Manipulative Mathematics

Using Manipulatives to Promote Understanding of Prealgebra Concepts

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To the Teacher:

Manipulatives give students concrete models of abstract mathematical concepts. Students using manipulatives develop conceptual understanding of arithmetic and algebraic topics and procedures because they see how and why the mathematical procedures work.

We created ***Manipulative Mathematics*** activities for those topics in ***Prealgebra*** where our teaching experience shows that developmental mathematics students have little understanding of mathematical concepts and tend to rely on poorly memorized procedures. The concrete manipulative activities help students visualize and understand the abstract arithmetic and algebraic concepts.

We have sets of manipulatives, such as **color chips**, **fraction pieces, algebra tiles**, **geoboards**, and more, in our classrooms. Our college students use them to model and learn about critical mathematical concepts and procedures. The degree to which we use manipulatives varies according to the needs of each class. Instead of lecturing on a topic, we guide our students through an activity in class to promote discovery learning. Later, we refer to the concrete activities when working examples on the board or answering student questions. We have heard many "a-ha's" from students who finally understand, for example, how to work with fractions or signed numbers.

You will find a reference to a ***Manipulative Mathematics*** activity in ***Prealgebra*** whenever a new arithmetic or algebraic concept is introduced. Illustrations of manipulatives are used to explain the concept and then students are encouraged to do a ***Manipulative Mathematics*** activity to help them develop a better understanding.

In this booklet, you will find a set of ***Manipulative Mathematics*** **packets** intended to serve as resources for teachers adopting ***Prealgebra***. Each packet includes one or more activities related to a mathematical topic covered in ***Prealgebra***.

Each activity includes an instructor page, student worksheet, and extra practice problems.

- The **instructor page** lists the resources needed, and gives information about the purpose of the activity and detailed directions for use in class. A link to an online source of virtual manipulatives similar to the physical ones used in the activity is provided on the instructor page of most activities.
- The **student worksheet** is a step-by-step developmental progression of questions leading students to discover a mathematical property or algorithm through the use of manipulatives.
- The **extra practice problems** reinforce the results of the student worksheet.

We have recorded eleven brief **Teacher Training Videos,** available on YouTube, to assist teachers in using the manipulatives featured in our ***Manipulative Mathematics*** activities. A link to the relevant video is shown on the cover page of each packet. Each video gives a brief overview of one type of manipulative, including:

- the math topic(s) it addresses
- a demonstration of teaching the topic(s) using the manipulative
- suggestions for using it in class
- where you can find it, in physical and virtual form

We sincerely hope our suite of instructor resources empower you to use our activities with your Prealgebra students! We recommend you start with just one activity the first time you use the book, and then include more activities in your course as you gain experience and fluency with manipulatives. We wholeheartedly believe that through the use of manipulatives, students develop an understanding of mathematics that translates into success in their subsequent courses.

Lynn Marecek MaryAnne Anthony-Smith

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Game of Twenty-four

Manipulatives used:

24®Game (optional)

Teacher Video: [Teaching The Order of Operations With The 24 Game]

Game of Twenty-four

Resources Needed:

Each student needs a worksheet. No other resources are required for this activity. The Game of Twenty-four is an old number game in which the players try to make 24 from a set of four numbers. A commercial version of the 24® Game, a boxed sets of cards with 4 numbers on each, grouped by difficulty level, is available at bookstores, teacher supply stores, game stores, and online at www.24game.com.

Background Information:

The Game of Twenty-four is a fun way to have students practice basic number facts, the order of operations, and correct algebraic notation, all critical to student success in Algebra. A good time to first use it is right after introducing the order of operations. Once students are familiar with the game, it can be used again at any time in your course.

To play the Game of Twenty-four, you start with a set of four numbers. The goal is to use each number once, and only once, with the four basic operations – addition, subtraction, multiplication, and division – to get 24. By creating their own expressions and verifying that they all give the answer 24, students apply the basic number facts and use the order of operations. The critical thinking process many students use to get 24 will help them throughout their math studies.

Directions:

- Explain the game to the students.

- Do one or more examples with the class, modeling the thought process you use to generate possible solutions. Students will usually volunteer solutions that use two numbers at a time then combine their 'partial answers' to get 24. Demonstrate how to write the entire solution as one mathematical expression that simplifies to 24. Emphasize the link between the order of operations and correct algebraic notation.

- Let the students do the worksheet, working alone or with a partner.

- A whole class discussion after students have completed the worksheets may be very valuable, especially for students who found this activity difficult. Successful students should be encouraged to share the strategies they used to get 24. Modeling the thought process involved can teach struggling students how to begin the critical thinking process.

Suggestions:

The Game of Twenty-four is a good way to use the extra couple of minutes that are sometimes overlooked in class. Put a set of four numbers on the board at the start of class and have students try to make 24 as you take attendance. Use it as an end of class activity when there is a little time left, but not enough to start a new concept or example. You might want to put a set of four numbers on a test or quiz as a bonus question.

Encourage your students to play the Game of Twenty-four at home. It can be played with children, ages 5 and older. There are several websites where you can play the Game of Twenty-four online. One of them is http://www.mathplayground.com/make_24.html. On that website you actually drag number cards and operation symbols into a workspace to make a mathematical expression.

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Game of Twenty-four

The Game of Twenty-four is a great way to think mathematically.
Given four numbers, you add, subtract, multiply and/or divide them so that the result is 24. You must use each number once--but only once.

Start with the numbers 1, 1, 4, and 8.

1) How can you use these numbers to create 24? Don't worry yet about 1, 1, 4, and 8. Think of pairs of any two numbers that multiply to 24. List some of the pairs here:

2) First, let's think of 24 as the product of $3 \bullet 8$.
   We want to combine **1, 1, 4, 8** to get 3 and 8.

   (a) One way is to use 4 minus 1 to get 3, then 3 times 8 is 24. But we need to use the number 1. How can we use the 1 and still have 24? 24 times 1 is still 24.

   Putting this all these steps together using good algebra notation gives $(4-1)(8)(1)$.
   Verify that this expression simplifies to 24.

   $$(4-1)(8)(1)$$

   (b) Here is another way to use the same four numbers, 1, 1, 4, 8, to get the product $3 \bullet 8$:
   4 times 1 is 4, and then 4 minus 1 gives 3. Finally multiply that 3 by 8 to get 24.
   Show that this expression simplifies to 24:

   $$(4 \bullet 1 - 1)(8)$$

3) This time, we'll use the fact that 24 is the product of $6 \bullet 4$.

   (a) Can we combine 1, 1, 4, 8 to make 6 times 4?
   Well, 1 plus 1 is 2, 8 minus 2 gives 6, and then 6 times 4 is 24.
   Show that this expression simplifies to 24:

   $$[8-(1+1)] \bullet 4$$

   (b) Can you think of another combination? Using good algebra notation, write a different expression and show that it simplifies to 24.

4) Another number fact that might help make 24 is $12 \bullet 2 = 24$.

(a) How can you combine 1, 1, 4, 8 to create 12 and 2?
4 plus 8 is 12, and 1 plus 1 is 2. Then twelve times two is 24!
Write this as one expression using good algebra notation, then show that it simplifies to 24.

(b) Can you think of another combination? Using good algebra notation, write a different expression and show that it simplifies to 24.

Now use the numbers 5, 3, 5, 4 to make 24.

5) Verify that each expression simplifies to 24.

(a) $5 \bullet 5 + 3 - 4$

(b) $(3 \bullet 5) + (5 + 4)$

6) Using good algebra notation, write a different expression that simplifies to 24.

Next try 3, 6, 6, 9.

7) Verify that each expression simplifies to 24.

(a) $3 + 6 + 6 + 9$

(b) $(6 \bullet 9) \div 3 + 6$

8) Using good algebra notation, write a different expression that simplifies to 24.

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Game of Twenty-four – Extra Practice

For each set of numbers use good algebra notation to write 2 different expressions that simplify to 24.

1) 1, 2, 3, 4

(a) (b)

2) 1, 2, 5, 9

(a) (b)

3) 1, 1, 7, 8

(a) (b)

4) 1, 7, 8, 9

(a) (b)

5) 2, 4, 6, 6

(a) (b)

6) 2, 3, 3, 6

(a) (b)

7) 2, 2, 4, 5

(a) (b)

8) 3, 3, 4, 5

(a) (b)

9) 3, 4, 5, 7

(a) (b)

10) 3, 4, 7, 9

(a) (b)

For more practice, there are several websites where you can play the Game of Twenty-four online. One of them is http://www.mathplayground.com/make_24.html.

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Number Lines

The Number Line Part 1-Counting Numbers and Whole Numbers
The Number Line Part 2-Integers
The Number Line Part 3-Fractions

Manipulatives used:

Paper tape
Markers

Teacher Video: Teaching Number Set Concepts

Number Line

Resources Needed:

Each student needs the worksheet, a red, blue, and black pen or marker and a three-foot strip of narrow paper. Adding machine paper works well.

Background Information:

By creating a number line and locating different sets of numbers on the line, students are making a concrete model of the number line. Students will visualize the relationship among number sets by outlining numbers with different symbols for each number set (counting, whole, integers). This kinesthetic activity accommodates the different learning styles of students. The number line model may also help students understand inequalities, especially when comparing negative numbers.

Directions:

Part 1-Counting numbers and whole numbers

- This activity may be done by individual students or together as a small group. Each student should create his own number line.
- Give each student the worksheet and a paper strip. Demonstrate folding the paper lengthwise and watch as the students fold theirs--there may be some confusion about how to fold the line. Then let the students follow the worksheet directions as you circulate through the classroom to offer individual guidance, as needed.
- Class discussion afterward will help reinforce the concepts of counting numbers and whole numbers.
- After students have completed this activity, tell them to save this number line in their notebooks, as they will use the same number line later to locate integers and fractions.

Part 2-Integers

- Integers will be added to the number line students created with counting numbers and whole numbers. Remind students to keep the unit the same for negatives as they used for positives.
- Then let the students proceed with the worksheet. Walk around the classroom to spot check student work.
- Class discussion afterward will help reinforce the concepts. Student number lines will now show counting numbers, whole numbers, and integers. While students have their number lines in front of them, you may wish to discuss ordering positive and negative numbers.
- After students have completed this activity, tell them to save this number line in their notebooks, as they will add fractions to this same number line later.

Part 3-Fractions

- After fractions have been introduced, they will be added to the number line. Many students seem to find this difficult.

- Demonstrate on the board how to divide the unit from 0 to 1 into various fractional units. Begin with halves and fourths. Continue, drawing a new unit interval for each new denominator, until the concept is clear.
- Then let the students proceed with the worksheet. Notice that the worksheet exercises do not use different denominators in the same unit interval. If there are student questions about locating, for example, $\frac{5}{3}$ when they already have $\frac{7}{4}$ marked (so the interval from 1 to 2 has been divided into fourths), draw a new number line.
- Class discussion afterward will help reinforce the concepts. Since students' number lines will have counting numbers, whole numbers, and integers marked, you may wish to introduce the term 'rational numbers' and explain how this number set relates to the others.
- Students can get additional practice locating fractions between 0 and 1 online at www.mathsisfun.com/numbers/fractions-match-frac-line.html. Given a fraction, students must decide how many intervals ('slices') to mark and then place the fraction at the correct mark. In addition to helping students locate proper fractions on the number line, it may also help students visualize fractions as parts of a whole.

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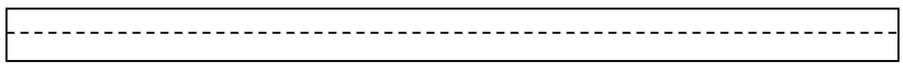
Number Line

The Number Line Part 1 -- Counting Numbers and Whole Numbers

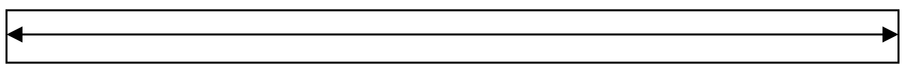
Counting numbers and whole numbers can be visualized by creating a **number line**.

1) To create your own number line:

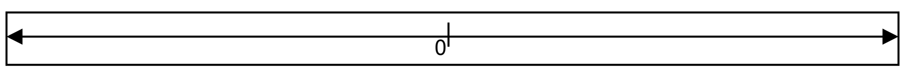
(a) Take a strip of paper about 3 feet long and fold it lengthwise to make a straight crease.



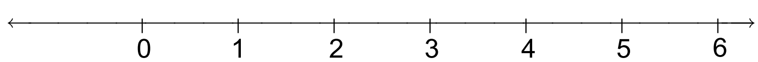
(b) Open the fold and draw a line in the crease. Put an arrow at each end of the line to indicate that the line continues.



(c) Mark a point at about the middle of the line. Label that point 0. This point is called the **origin**.

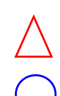


2) Choose a convenient unit and mark off several of these units to the right of 0. Pair these points with the numbers 1, 2, 3, 4, 5, …..and so on. When a number is paired with a point, we call it the **coordinate** of the point.



3) Draw a red triangle around each counting number.

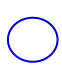
4) Draw a blue circle around each whole number.



5) Notice that all the numbers on your number line except 0 are marked with both a triangle and a square. What conclusion can you draw from this?

6) In one corner of your strip make a "key" that explains the symbols around the numbers.

Counting numbers



Whole numbers

7) Put your number line in your notebook for future use, so you can add more numbers to the number line as you proceed through this course.

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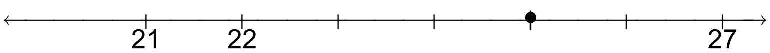
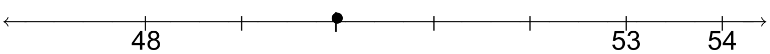
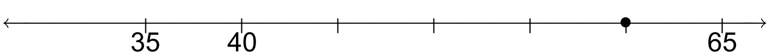
Number Line Part I – Extra Practice

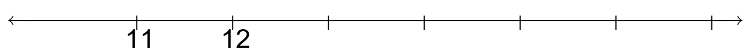
Name the coordinate of each point.

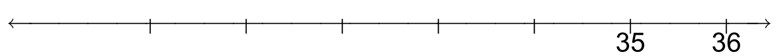
1) 

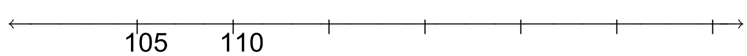
2) 

3) 

Locate each point on the number line.

4) 15 

5) 31 

6) 125 

For each set of numbers identify (a) the counting numbers and (b) the whole numbers.

7) $0, \frac{1}{5}, 4, 7.5, 23, 199$ (a) ________________ (b) ________________

8) $0, \frac{3}{4}, 1, 5\frac{1}{2}, 16, 99.9, 250$ (a) ________________ (b) ________________

9) $0, \frac{2}{9}, 3.1, 6, 10\frac{1}{4}, 88, 132.5$ (a) ________________ (b) ________________

10) $0, 1, \frac{5}{2}, 5.2, 8, 24.99, 165, 200$ (a) ________________ (b) ________________

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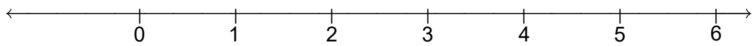
Number Line

The Number Line Part 2 -- Integers

The number line you made in Part 1 started at 0. All the numbers you have worked with so far have been positive numbers, numbers greater than 0.

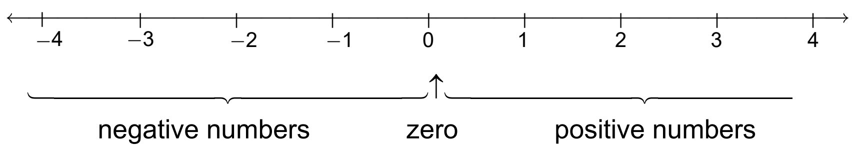


Now you need to expand your number line to include negative numbers, too. Negative numbers are numbers less than zero. So the negative numbers will be to the left of zero on the number line.

Get your number line out of your notebook and place it on your desk.

1) Mark off several units to the **left** of zero. Make sure your unit is the same size as the one you used on the positive side.

2) Now label -1 at the first point left of 0, then -2 at the next point to the left, and so on.

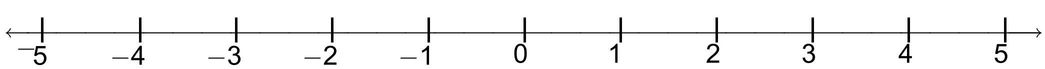


3) The arrows on both ends of the number line indicate that the numbers keep going forever.
(a) Is there a largest positive number? ____ (b) Is there a smallest negative number? ____

4) Is zero a positive or a negative number?______________ Numbers larger than zero are positive and numbers smaller than zero are negative. Zero is neither positive nor negative.

5) Locate and label the following points on this number line.

(a) 2 (b) -1 (c) -4 (d) 5 (e) -5



The non-zero whole numbers and their opposites, as well as zero, are called the **integers**.

Integers: $\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$

6) Put a black square around each integer on your number line.

7) What do you notice about the integers, counting numbers and whole numbers on your number line?

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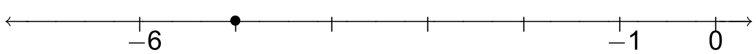
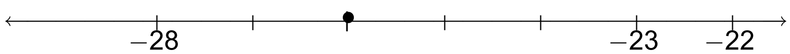
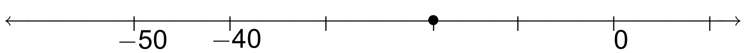
Number Line Part 2 – Extra Practice

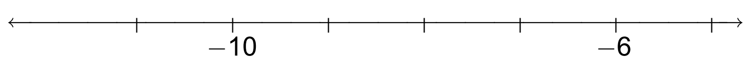
Name the coordinate of each point.

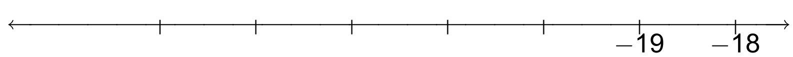
1) 

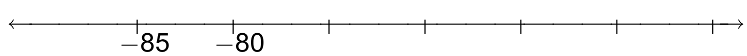
2) 

3) 

Locate each point on the number line.

4) -9 

5) -22 

6) -65 

For each set of numbers identify the (a) counting numbers, (b) whole numbers, and (c) integers.

7) $-3,\ -\frac{1}{2},\ 0,\ \frac{9}{10},\ 5,\ 7.5,\ 32$ (a) __________ (b) __________ (c) __________

8) $-12,\ -\frac{3}{4},\ 0,\ 2,\ 4.65,\ 29,\ 48\frac{1}{6}$ (a) __________ (b) __________ (c) __________

9) $-8.2,\ -3,\ -\frac{5}{9},\ 0,\ 4,\ \frac{26}{3},\ 99$ (a) __________ (b) __________ (c) __________

10) $-\frac{15}{4},\ -2.5,\ -1,\ 0,\ \frac{4}{7},\ 10,\ 28.1$ (a) __________ (b) __________ (c) __________

Manipulative Mathematics **Name______________________**

Number Line

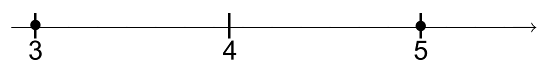
The Number Line Part 3 -- Fractions

Now you are ready to include fractions on your number line. This will help you visualize fractions and understand their value. Take your number line out of your notebook and place it on your desk.

Our goal is to locate the numbers $\frac{1}{5}$, $\frac{4}{5}$, 3, $3\frac{1}{3}$, $\frac{7}{4}$, $\frac{9}{2}$, 5, and $\frac{8}{3}$ on the number line.

1) We'll start with the whole numbers 3 and 5 because they are the easiest to plot.

   Put points to mark 3 and 5.



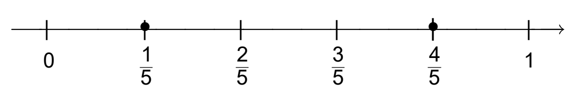
2) The proper fractions listed are $\frac{1}{5}$ and $\frac{4}{5}$.

   (a) Proper fractions have value less than one. Between which two whole numbers are the proper fractions $\frac{1}{5}$ and $\frac{4}{5}$ located? They are between _____ and _____.

   (b) Their denominators are both 5.
   So into how many pieces do you need to divide the unit from 0 to 1?_____
   How many marks will you need to divide the unit into that many pieces? _____

   (c) Divide the unit from 0 to 1 into five equal parts, and label the marks, consecutively, $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$.

   (d) Now put points to mark $\frac{1}{5}$ and $\frac{4}{5}$.

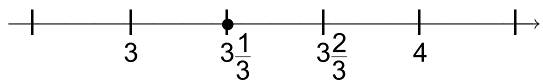


3) The only mixed number to plot is $3\frac{1}{3}$.

   (a) Between which two whole numbers is $3\frac{1}{3}$? Remember that a mixed number is a whole number plus a proper fraction, so $3\frac{1}{3} > 3$. Since it is greater than three, but not a whole unit greater, $3\frac{1}{3}$ is between _____and _____.

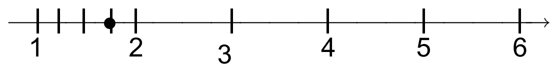
   (b) Divide that portion of the number line into _____ equal pieces (thirds) by making ______ marks.

(c) Plot $3\frac{1}{3}$ at the first mark.

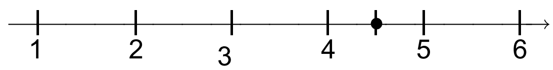


4) Finally, look at the improper fractions $\frac{7}{4}, \frac{9}{2}, \frac{8}{3}$. Locating these points will be easier if you change each of them to a mixed number.

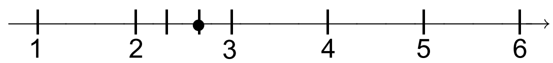
(a) $\frac{7}{4} =$ ______



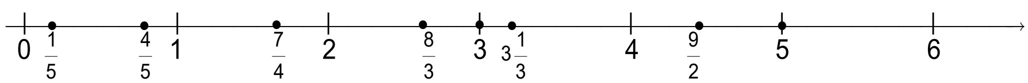
(b) $\frac{9}{2} =$ ______



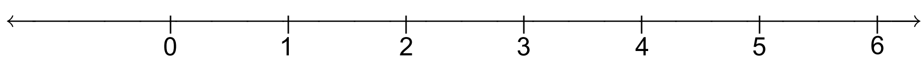
(c) $\frac{8}{3} =$ ______



5) Here is the number line with all the points ($\frac{1}{5}$, $\frac{4}{5}$, 3, $3\frac{1}{3}$, $\frac{7}{4}$, $\frac{9}{2}$, 5, and $\frac{8}{3}$) plotted. Verify that your number line looks the same.



6) Locate and label the fractions $\frac{3}{4}$, $\frac{4}{3}$, $\frac{5}{3}$, $4\frac{1}{5}$, $\frac{7}{2}$ on the number line below.



Now let's locate some negative fractions.

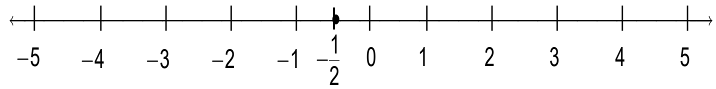
7) We'll locate $-\frac{1}{2}$ first. Remember that negative numbers are opposites of positive numbers, so $-\frac{1}{2}$ is the opposite of $\frac{1}{2}$.

(a) Since $\frac{1}{2}$ is between the two whole numbers _______ and _______, $-\frac{1}{2}$ is between the two integers _______ and ___________.

(b) Into how many pieces do we need to divide the unit between 0 and -1? ________

(c) Divide that portion of the number line into _____ equal pieces (halves) by making ______ marks.

(d) Plot $-\frac{1}{2}$ at the mark.



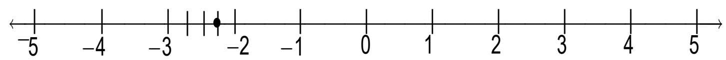
8) Now let's locate $-2\frac{1}{4}$ on a number line.

(a) Think about $2\frac{1}{4}$ first. It is located between the whole numbers ______ and ______.

(b) So $-2\frac{1}{4}$ is between ________ and ________.

(c) Into how many equal pieces do we need to divide that unit? _________

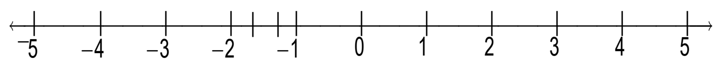
(d) Plot $-2\frac{1}{4}$ at the first mark.



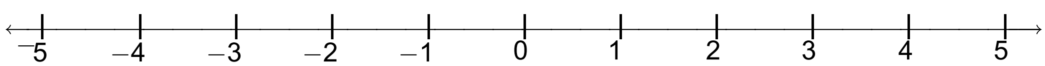
9) Locating $-\frac{5}{3}$ on a number line will be easier if you first change it to a mixed number.

(a) $-\frac{5}{3}$ = ___________. It is between _________ and _________.

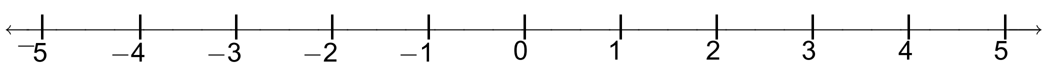
(b) Plot $-\frac{5}{3}$.



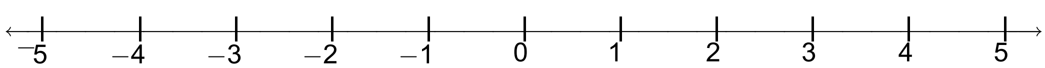
10) Locate and label the fractions $\frac{2}{3}$ and $-\frac{2}{3}$ on the number line below.



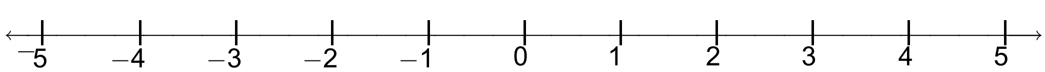
11) Locate and label the fractions $-\frac{9}{2}$ and $-\frac{9}{4}$ on the number line below.



12) Locate and label the fractions $\frac{7}{3}$, $-3\frac{3}{4}$, $3\frac{1}{3}$, and $-\frac{8}{5}$ on the number line below.



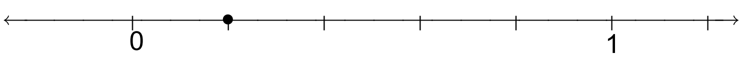
13) Locate and label the fractions $\frac{1}{3}$, $-\frac{5}{4}$, $-\frac{7}{4}$, $2\frac{3}{5}$, and $-3\frac{1}{2}$ on the number line below.

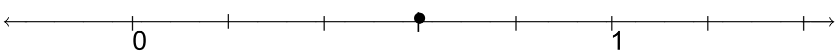


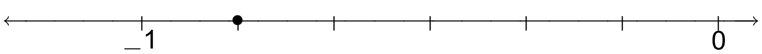
Manipulative Mathematics **Name**______________________

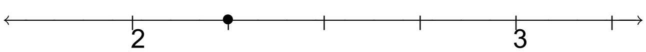
Number Line Part 3 – Extra Practice

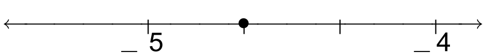
Name the coordinate of each point.

1) 

2) 

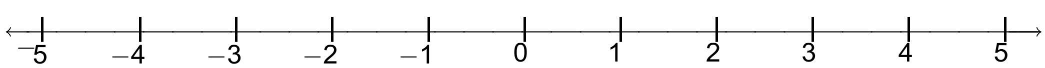
3) 

4) 

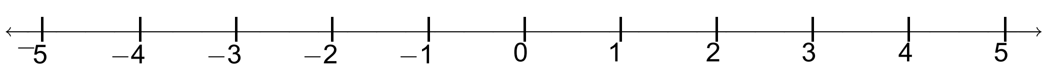
5) 

Locate and label each point on the number line.

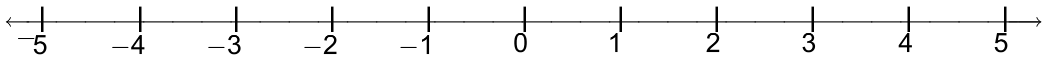
6) (a) $\frac{3}{5}$ (b) $1\frac{2}{3}$ (c) $-\frac{9}{4}$



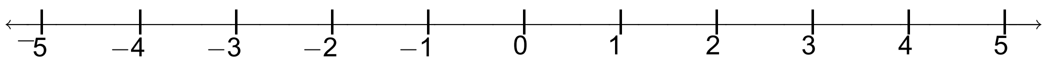
7) (a) $\frac{10}{3}$ (b) $-2\frac{4}{5}$ (c) $-\frac{1}{2}$



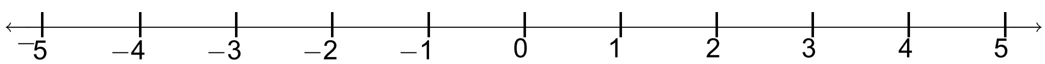
8) (a) $-3\frac{3}{4}$ (b) $\frac{1}{3}$ (c) $\frac{15}{4}$



9) (a) $-\frac{5}{4}$ (b) $4\frac{2}{3}$ (c) $-\frac{4}{5}$



10) (a) $\frac{5}{8}$ (b) $-3\frac{1}{4}$ (c) $-\frac{14}{3}$



You can do more practice locating fractions on the number line at the website:
http://www.mathsisfun.com/numbers/fractions-match-frac-line.html.

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Multiples and Factors

Multiplication/Factors

Square Numbers

Manipulatives used:

Square ‘color’ tiles

Counters

Teacher video: [Teaching Multiples and Factors]()

Multiplication/Factors

Resources Needed:

Each student needs a set of 25 square tiles (often called 'color tiles') and a sheet of graph paper.

Background Information:

Many students have a limited visual understanding of multiplication. This activity is designed to give them a visual representation of multiplication as they build rectangular arrays. These arrays and activities will help them understand factors, the commutative property of multiplication, and prime numbers.

Directions:

- This exercise usually works best with students working in pairs or groups. Give each student a set of 25 color tiles.

- Work exercise 1 together with the class. Show your students how you can make one rectangle using 12 tiles, and then ask them to arrange 12 of their tiles into another rectangle. Ask the students to describe their rectangles, and draw and label each one. Continue asking students for more rectangles until all 3 unique combinations (1x12, 2x6, 3x4) are displayed. As you label each rectangle with its dimensions (ie, 2x6), explain that a 2x6 and a 6x2 (etc.) rectangle are really the same. You may want to rotate or superimpose a model to show this. Have the students color a representation of the three possible rectangles on their graph paper and label each '12'.

- Then have students proceed through the worksheet. They will find all possible rectangles using exactly 1 tile, 2 tiles, 3 tiles, …, 25 tiles. Students will draw their rectangles on their graph paper and label the rectangles with their dimensions.

- These rectangles and the summary chart can lead into rich discussions about factors, primes, multiples, square numbers, etc. Allow time for discussion to develop in the groups and then bring the class together for further discussion.

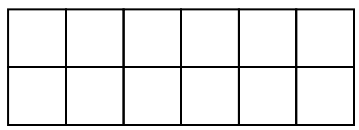
- You may want to demonstrate exercise 1 by using the square pattern blocks at http://nlvm.usu.edu/en/nav/frames_asid_169_g_1_t_3.html?open=activities&from=topic_t_3.html.

Manipulative Mathematics Name______________________

Multiplication/Factors

1) Take twelve tiles and form a rectangle.
   (a) How many rows does your rectangle have? _____
   (b) How many columns? _____
   (c) Each rectangle can be called a $\underbrace{\qquad\qquad}_{\text{number of rows}} \times \underbrace{\qquad\qquad}_{\text{number of columns}}$ rectangle.

   The number of rows and the number of columns are called the **dimensions** of the rectangle.



____rows by ____columns

____ $\times$ ____ rectangle

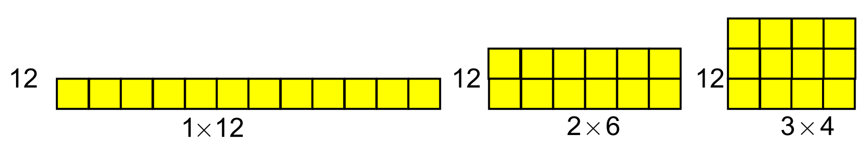
   (d) How many tiles were used to make this rectangle? ____
   (e) What is the product of $2 \bullet 6$? ____
   (f) Form a 6×2 rectangle. Draw it here:

   (g) What do you notice about the 2×6 and the 6×2 rectangles?

   A 2×6 rectangle and a 6×2 rectangle are **equivalent**. This means you could rotate the 2×6 rectangle and it would look exactly the same as the 6×2 rectangle.

2) Now, create all possible rectangles using 1 tile, 2 tiles, 3 tiles, …, 25 tiles.
   (a) Copy each rectangle onto graph paper.
   (b) Label each rectangle with the total number of tiles used to form it.
   (c) Under the rectangle write its dimensions: $\underbrace{\qquad\qquad}_{\text{number of rows}} \times \underbrace{\qquad\qquad}_{\text{number of columns}}$.

   For example, your graph paper would show 3 rectangles for 12 tiles:



(d) Summarize your results in the chart below.

Number of tiles	Dimensions of the rectangles formed
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	1×12, 2×6, 3×4
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	

Use your chart to answer the following questions.

3) Look for all the rectangles in your chart that have 2 rows.

(a) List the dimensions of all the rectangles that have 2 rows.

2×1, 2×2, ______, ______, ______, ______, ______, ______, ______, ______, ______, ______

(b) Now list the total number of tiles you used to form each rectangle you listed in 5.

2, 4, ____, ____, ____, ____, ____, ____, ____, ____, ____, __

These numbers are called the **multiples** of 2.

Multiple
A number is a multiple of n if it is the product of a counting number and n.

4) How can you use your rectangle chart to find the multiples of 3?

5) List the multiples of three: 3, ___, ___, ___, ___, ___, ___, ___.

6) List the multiples of four: 4, ___, ___, ___, ___, ___.

7) List the multiples of five: 5, ___, ___, ___, ___.

Notice that with 12 tiles, we could form 3 different rectangles, 1×12, 2×6, and 3×4. The numbers 1, 2, 3, 4, 6, and 12 are **factors** of 12, because $1 \bullet 12 = 12$, $2 \bullet 6 = 12$, and $3 \bullet 4 = 12$.

Factors

If $a \bullet b = m$, then a and b are factors of m.

8) List all the factors of 15: ___, ___, ___, ___

9) Which number from 1 to 25 has the most factors? ___________

10) Which number of tiles can be used to make the most rectangles? ___________

11) Explain why some numbers can be used to make more rectangles than other numbers.

12) List the numbers for which you could only form one rectangle.

These numbers are called **primes**. A prime number has only two factors, 1 and itself.

Prime

A prime number is a counting number greater than 1, whose only factors are 1 and itself.

13) List all the primes between 2 and 25.

14) What other number relationships do you notice in your rectangle chart?

Manipulative Mathematics **Name______________________**

Multiplication/Factors – Extra Practice

List the first ten multiples of the following numbers.

1) 6: ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

2) 7: ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

3) 8: ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

4) 9: ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

5) 12: ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

The number 16 can be factored $1 \bullet 16$, $2 \bullet 8$, and $4 \bullet 4$, so all the factors of 16 are 1, 2, 4, 8, and 16. Find all the factors of each of the following numbers.

6) 24 ______________________________

7) 30 ______________________________

8) 42 ______________________________

9) 63 ______________________________

10) 135 ______________________________

Manipulative Mathematics **Instructor Page**

Square Numbers

Resources Needed:

Each student needs a worksheet and about 50 counters.

Background Information:

This activity is designed to give students a visual representation of the square of a number. While this exercise may seem almost trivial to a teacher, it can be difficult for some students. They may never have realized how the word ‘square’ in 3^2 relates to a shape. Usually, once students begin to see the concept of square, they make the connection between the numbers and the shapes. The rewards of this activity validate the time spent. Students are able to identify perfect squares more easily, which is a great help as they begin factoring.

Directions:

- This activity is best done by students working individually at the start. Students will need a partner to do Exercise 4.
- Give each student a worksheet and about 50 counters. (50 counters will allow them to model square numbers through $7^2 = 49$.)
- Model Exercise 1a for the class. Show students the concept of the square number 4 using counters.
- Have the students continue through the worksheet. When they get to the Exercise 4, they will need to work with a partner.
- When most students have completed the activity, bring the class together for discussion. You may want to review the definition of square numbers shown above Exercise 5. If you have students in your class who speak languages other than English, you may want to ask them how to say, for example, 7^2 and the shape ‘square’ in their language. Chances are they are both the same word!
- You may want to demonstrate exercise 1 by using the square pattern blocks at http://nlvm.usu.edu/en/nav/frames_asid_169_g_1_t_3.html?open=activities&from=topic_t_3.html.

Manipulative Mathematics **Name_________________________**

Square Numbers

1) Put about 50 color counters on your workspace. We will use the counters to make squares.
   (a) For example,  is a square made from ___ counters. It has ______counters on each side.

   (b) Make as many squares as you can with your counters. Draw a picture of each square that you create and record your results in the table below:

Picture of square							
Total number of counters in the square		4					
Number of counters on each side		2					

2) Can you make a square with exactly 6 counters? ______ Why or why not?

3) Imagine if you had 100 counters
   (a) Could you make a square with exactly 100 counters? ______
   (b) Why or why not?

   (c) How many counters would be on each side of a square made with100 counters? ____

4) Work with a partner and put all your counters together.
   (a) Create a square that uses more than 50 counters. Draw a sketch of your square.

(b) Create all the squares you can using 50 to 100 counters. Sketch your squares here.

When a number n is multiplied by itself, we write it n^2 and read it 'n squared'. For example,

8^2 is read '8 squared'.

64 is called 'the square of 8'.

Similarly, 121 is the square of 11, because 11^2 is 121.

Square of a number

If $n^2 = m$, then m is the square of n.

5) Complete this table to show the squares of the counting numbers 1 through 15.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n^2								64			121				

The squares of the counting numbers are called **perfect squares**, so the second row of the table shows the first fifteen perfect squares.

6) List the total number of counters you used for each square you made in Exercise 1(b).

7) Do you see a similarity between the table you filled in for Exercise 1 with the pictures of squares and the table you made in Exercise 5 with the squares of the counting numbers 1 through 15?___

(a) Describe how the two tables are alike.

(b) Why do we use the word 'square' for both the symbol in 3^2 and the shape $\square$?

Manipulative Mathematics **Name______________________**

Square Numbers – Extra Practice

Identify whether each number is a perfect square. If it is a perfect square, write is as the square of a counting number.

Number	Not a perfect square	Yes – perfect square
1) 36	______	$\_\_\_\_\_\_ = (\_\_\_\_\_)^2$
2) 50	______	$\_\_\_\_\_\_ = (\_\_\_\_\_)^2$
3) 140	______	$\_\_\_\_\_\_ = (\_\_\_\_\_)^2$
4) 196	______	$\_\_\_\_\_\_ = (\_\_\_\_\_)^2$
5) 221	______	$\_\_\_\_\_\_ = (\_\_\_\_\_)^2$
6) 289	______	$\_\_\_\_\_\_ = (\_\_\_\_\_)^2$
7) 364	______	$\_\_\_\_\_\_ = (\_\_\_\_\_)^2$
8) 625	______	$\_\_\_\_\_\_ = (\_\_\_\_\_)^2$
9) 784	______	$\_\_\_\_\_\_ = (\_\_\_\_\_)^2$
10) 961	______	$\_\_\_\_\_\_ = (\_\_\_\_\_)^2$

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Visualizing Fractions

Model Fractions
Fractions Equivalent to One
Mixed Numbers and Improper Fractions
Equivalent Fractions

Manipulatives used:

Fraction circles
Fraction tiles

Teacher video: Teaching Fraction Concepts

Model Fractions

Resources Needed:

Each student needs a worksheet, a set of fractions tiles, and a set of fraction circles.

Background Information:

Fractions are a very abstract idea to many students at this level. Students don't have a concrete model of fractions they can relate to, and so working with fractions becomes mere manipulation of symbols for no apparent reason. This activity helps students make the connection between a concrete fraction and the abstract concepts and symbols. The activity takes very little time, but the rewards are great.

Directions:

- In this quick activity the meaning of the numerator and denominator in a fraction are shown to correspond to parts of a whole. Students can complete this worksheet without using manipulatives. It is best done individually.
- Give each student a worksheet.
- Demonstrate for the class one "set" of fraction circles and one "set" of fraction tiles, (i.e., 3 thirds, 4 fourths, etc.). Show and explain how, for example, $\frac{1}{3}$ means 1 of the 3 equal pieces that together make one whole, and $\frac{2}{3}$ represents 2 of those pieces. Emphasize the meaning of fractions as parts of a whole.
- Have the students proceed through the worksheet on their own.
- When most students seem to have completed the worksheet, bring the class together again for discussion.
- Students can get additional practice naming fractions online at the National Library of Virtual Manipulatives website:
  - Fractions – parts of a whole
    http://nlvm.usu.edu/en/nav/frames_asid_102_g_2_t_1.html?from=topic_t_1.html
  - Fractions – naming
    http://nlvm.usu.edu/en/nav/frames_asid_104_g_1_t_1.html?from=topic_t_1.html
  - Fraction pieces
    http://nlvm.usu.edu/en/nav/frames_asid_274_g_3_t_1.html?open=activities&from=topic_t_1.html

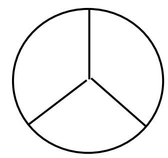
Manipulative Mathematics **Name**______________________

Model Fractions

> **Fraction:**
> A fraction is written $\frac{a}{b}$
> a is the **numerator** and b is the **denominator**.

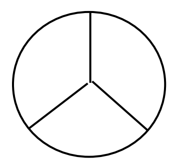
Fractions are a way to represent parts of a whole. The fraction $\frac{1}{3}$ means that one whole has been divided into 3 equal parts and each part is one of the three equal parts.

1) This circle that has been divided into 3 equal parts. Label each part $\frac{1}{3}$.

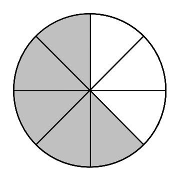


2) What does the fraction $\frac{2}{3}$ represent? This means the whole has been divided into 3 equal parts, and $\frac{2}{3}$ represents two of those three parts.

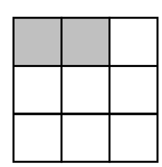
   Shade two out of the three parts of this circle to represent $\frac{2}{3}$.



3) What fraction of this circle is shaded?
   (a) How many parts are shaded? ______
   (b) How many equal parts are there? ______
   (c) The fraction of the circle that is shaded is $\frac{\square}{\square}$.

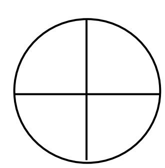


4) What fraction of this square is shaded?
   (a) How many parts are shaded? _____
   (b) How many equal parts are there? _____
   (c) The fraction of the square that is shaded is $\frac{\square}{\square}$



5) To shade — of the circle, shade ______ out of the ______ parts.

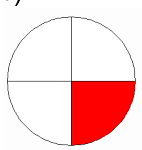
   Shade $\frac{3}{4}$.



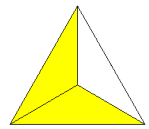
Manipulative Mathematics **Name______________________**

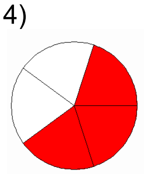
Model Fractions – Extra Practice

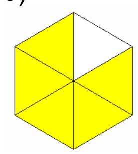
Name the fraction modeled by each figure.

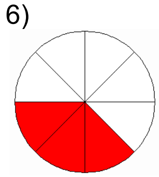
1)  ________

2)  ________

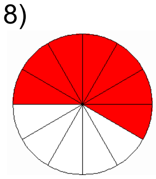
3)  ________

4)  ________

5)  ________

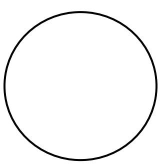
6)  ________

7)  ________

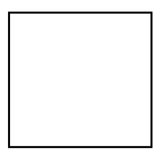
8)  ________

Model each fraction.

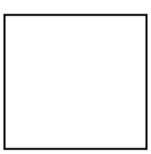
9) $\frac{1}{6}$



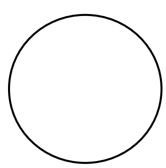
10) $\frac{5}{9}$



11) $\frac{4}{5}$



12) $\frac{7}{8}$



For more practice

- naming fractions, go to:
  http://nlvm.usu.edu/en/nav/frames_asid_104_g_1_t_1.html?from=topic_t_1.html
- modeling fractions, go to:
  http://nlvm.usu.edu/en/nav/frames_asid_102_g_2_t_1.html?from=topic_t_1.html

Fractions Equivalent to One

Resources Needed:

Each student needs a worksheet and a set of fractions tiles.

Background Information:

Fractions are a very abstract idea to many students at this level. Students don't have a concrete model of fractions they can relate to, and so working with fractions becomes mere manipulation of symbols for no apparent reason. This activity helps students make the connection between a concrete fraction and the abstract concepts and symbols. The activity takes very little time, but the rewards are great.

Directions:

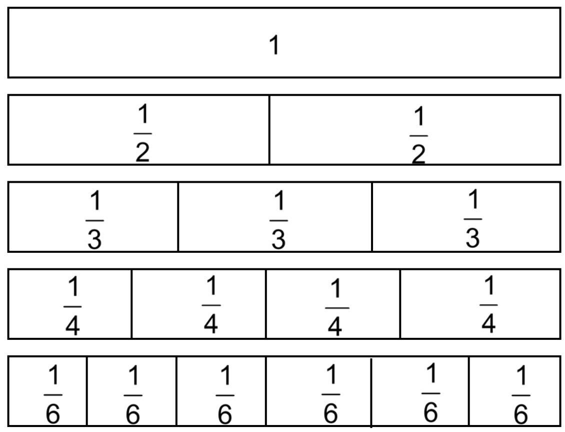
- In this activity students use fraction tiles to model fractions equivalent to one. Students may work individually or with partners.
- Give each student a set of fraction tiles and a worksheet.
- Demonstrate for the class how to put all the fraction tiles together to make a rectangle of width one.

- Have the students proceed through the worksheet on their own or in their groups. Some students may need clarification when they attempt to answer the questions.
- When most students seem to have completed the worksheet, bring the class together again for discussion. You may want to ask the students for their answers to Exercise 5 and then list the 'patterns' they described in Exercise 6.
- The interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html shows a set of fraction tiles. Students can use it to verify, for example, that it takes fourteen $\frac{1}{14}$ pieces to make one.

Manipulative Mathematics **Name___________________**

Fractions Equivalent to One

Fractions are often shown as parts of rectangles. Here, the whole is one long rectangle.



Set up your fraction tiles as shown in the diagram above.

1) How many of the $\frac{1}{2}$ tiles does it take to make 1 whole tile?

(a) It takes _____halves to make a whole. (b) Two out of two is 1 whole. $\frac{2}{2} =$ ____ .

2) How many of the $\frac{1}{3}$ tiles does it take to make 1 whole tile?

(a) It takes _____ thirds to make a whole. (b) Three out of three is 1 whole.
$\frac{3}{3} =$ ____.

3) How many of the $\frac{1}{4}$ tiles does it take to make 1 whole tile?

(a) It takes _____fourths to make 1 whole. (b) Four out of four is 1 whole. $\frac{4}{4} =$ ____.

4) How many of the $\frac{1}{6}$ tiles does it take to make 1 whole tile?

(a) It takes ______ sixths. (b) Six out of six is 1 whole. $\frac{6}{6} =$ _____ .

5) What if the whole was divided into 24 equal parts? We don't have fraction tiles to represent this and it is too many to draw easily, but try to visualize it in your mind.

(a) How many $\frac{1}{24}$'s does it take to make 1? ______ (b) $\frac{\square}{24} = 1$

6) Do you see any pattern here? Describe the pattern you see.

Manipulative Mathematics **Name**______________________

Fractions Equivalent to One – Extra Practice

Use fraction tiles to answer these exercises.

You may want to use virtual fraction tiles on the interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html.

1) How many $\frac{1}{5}$'s does it take to make 1? ______________

2) How many $\frac{1}{8}$'s does it take to make 1? ______________

3) How many $\frac{1}{10}$'s does it take to make 1? ______________

4) How many $\frac{1}{13}$'s does it take to make 1? ______________

5) How many $\frac{1}{16}$'s does it take to make 1? ______________

6) How many $\frac{1}{32}$'s does it take to make 1? ______________

7) Fill in each numerator.

(a) $\frac{\square}{9}=1$ (b) $\frac{\square}{12}=1$ (c) $\frac{\square}{14}=1$

8) Fill in each denominator.

(a) $\frac{8}{\square}=1$ (b) $\frac{11}{\square}=1$ (c) $\frac{15}{\square}=1$

9) Fill in the missing part.

(a) $\frac{\square}{7}=1$ (b) $\frac{20}{20}=\square$ (c) $\frac{25}{\square}=1$

(d) $\frac{41}{41}=\square$ (e) $\frac{64}{\square}=1$ (f) $\frac{\square}{100}=1$

Manipulative Mathematics **Instructor Page**

Mixed Numbers and Improper Fractions

Resources Needed:

Each student needs a worksheet and a set of fraction circles.

Background Information:

Fractions are a very abstract idea to many students at this level. Students don't have a concrete model of fractions they can relate to, and so working with fractions becomes mere manipulation of symbols for no apparent reason. These activities help students make the connection between a concrete fraction and the abstract concepts and symbols. The activities takes very little time, but the rewards are great.

Directions:

- In this activity students use fraction circles to model improper fractions and mixed numbers. Students should work in pairs, so they can model fractions larger than one.
- Give each student a set of fraction circles and a worksheet. Even though they work with a partner, all students should complete their own worksheets.
- Have the students work through the worksheet with their partners. Some students may need a hint to draw a third circle for question 2d.
- When most students seem to have completed the worksheet, bring the class together again for discussion. You may want to have the students share their answers to questions 8 and 11.
- Students can get more practice modeling improper fractions and visualizing how to convert between improper fractions and mixed numbers at the National Library of Virtual Manipulatives website.
    - Fraction pieces
      http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&from=topic_t_1.html

Manipulative Mathematics **Name**_____________________

Mixed Numbers and Improper Fractions

1) Use fraction circles to make wholes, if possible, with the following pieces. Draw a sketch to show your result.
   (a) 2 halves (b) 6 sixths (c) 4 fourths (d) 5 fifths

2) Use fraction circles to make wholes, if possible, with the following pieces. Draw a sketch to show your result.
   (a) 3 halves (b) 5 fourths

   (c) 8 fifths (d) 7 thirds

When a fraction has the numerator smaller than the denominator, it is called a **proper** fraction. Its value is less than one. Fractions like $\frac{1}{2}$, $\frac{3}{7}$, and $\frac{11}{18}$ are proper fractions.

A fraction like $\frac{5}{4}$, $\frac{3}{2}$, $\frac{8}{5}$, or $\frac{7}{3}$ is called an **improper** fraction. Its numerator is greater than its denominator. Its value is greater than one.

Proper and Improper Fractions
The fraction $\frac{a}{b}$ is: $(b \neq 0)$
proper if $a < b$ or **improper** if $a \geq b$

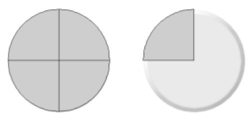
3) Write as improper fractions.
   (a) 3 halves_____ (b) 5 fourths _____ (c) 8 fifths_____ (d) 7 thirds_____

4) Look back at your models in Exercise 2 and the improper fractions in Exercise 3. Which improper fraction in Exercise 3 could also be written as $1\frac{1}{4}$? ________

The number $1\frac{1}{4}$ called a **mixed number**; it consists of a whole number and a proper fraction.

> **Mixed Number**
>
> A **mixed number** is written $\quad a\frac{b}{c} \qquad c \neq 0$
>
> A mixed number consists of a whole number a and a proper fraction $\frac{b}{c}$.

The model shows that $\frac{5}{4}$ has the same value as $1\frac{1}{4}$.



$$\frac{5}{4} = 1\frac{1}{4}$$

5) Write each improper fraction as a mixed number. You may want to refer to your models in Exercise 2.

(a) $\frac{3}{2}$ _____ (b) $\frac{5}{4}$ _____ (c) $\frac{8}{5}$ _____ (d) $\frac{7}{3}$ _____

6) Rewrite the improper fraction — as a mixed number. Use fraction circles to find the result.

(a) Draw a sketch to show your answer. (b) $\frac{11}{6} =$ _____

7) Rewrite the improper fraction — as a mixed number. Use fraction circles to find the result.

(a) Draw a sketch to show your answer. (b) $\frac{17}{5} =$ _____

8) Explain how you convert an improper fraction as a mixed number.

9) Rewrite the mixed number $1\frac{2}{3}$ as an improper fraction.

(a) Draw a sketch to show your answer. (b) $1\frac{2}{3} =$ _____

10) Rewrite the mixed number $2\frac{1}{4}$ as an improper fraction.

(a) Draw a sketch to show your answer. (b) $2\frac{1}{4} =$ _____

11) Explain how you convert a mixed number to an improper fraction.

Manipulative Mathematics **Name**_____________________

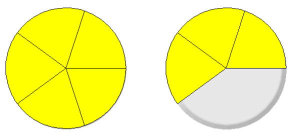
Mixed Numbers and Improper Fractions – Extra Practice

Use 2 sets of fraction circles to do these exercises.

You may want to use the fraction circles on the interactive website:
http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&from=topic_t_1.html.

Name each improper fraction. Then write each improper fraction as a mixed number.

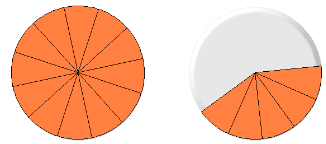
1)



(a) improper fraction ____________

(b) mixed number ____________

2)



(a) improper fraction ____________

(b) mixed number ____________

Draw a figure to model the following improper fractions. Then write each as a mixed number.

Improper fraction	Model	Mixed number
3) $\frac{7}{4}$		$\frac{7}{4} =$
4) $\frac{9}{5}$		$\frac{9}{5} =$
5) $\frac{17}{10}$		$\frac{17}{10} =$
6) $\frac{10}{3}$		$\frac{10}{3} =$

Draw a figure to model the following mixed numbers. Then write each as an improper fraction.

Mixed number	Model	Improper fraction
7) $1\frac{2}{5}$		$1\frac{2}{5} =$
8) $1\frac{1}{6}$		$1\frac{1}{6} =$
9) $1\frac{7}{12}$		$1\frac{7}{12} =$
10) $2\frac{3}{4}$		$2\frac{3}{4} =$

Equivalent Fractions

Resources Needed:

Each student needs a worksheet and a set of fractions tiles.

Background Information:

Many students that take this course have never really understood fractions. Often they just manipulate the symbols without any thoughts about their meaning, and as a result are just as likely to apply an incorrect procedure as the correct one. This activity helps students understand the concept of equivalent fractions and the procedure to find them; students will see how the abstract concepts and symbols relate to the concrete fraction tiles. This worksheet takes very little time, but the rewards are great.

Directions:

- Students may do this activity individually or in a small group.

- Give each student a set of fraction tiles and a worksheet. Be sure they all have an adequate amount of clear desk space to set out their fraction tiles.

- Guide them through the first part of the activity – finding how many fourths equal one-half. You may wish to use fraction tiles with a projector to demonstrate what it means to "exactly cover" the one-half tile.

- Let students continue with the worksheet on their own or in their groups.

- Discussion at the end will help reinforce the concepts. You may want to have students explain their answers to Exercises 6, 9, 13, and 14.

- The interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html shows a set of fraction tiles. Students drag a line across the set of tiles to see all the equivalent fractions.

Manipulative Mathematics **Name____________________**

Equivalent Fractions

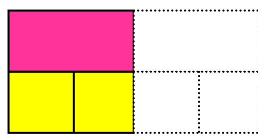
Equivalent Fractions
Equivalent fractions have the same value.

Use fraction tiles to do the following activity:

1) Take one of the $\frac{1}{2}$ tiles and set it on your workspace.

(a) How many fourths equal one-half?

Take the $\frac{1}{4}$ tiles and place them below the $\frac{1}{2}$ tile.



How many of the $\frac{1}{4}$ tiles exactly cover the $\frac{1}{2}$?______

(b) Since ______ of the $\frac{1}{4}$ tiles cover the $\frac{1}{2}$ tile,

we see $\frac{\square}{4}$ is the same as $\frac{1}{2}$. $\qquad \frac{\square}{4} = \frac{1}{2}$

2) How many sixths equal one-half?

(a) How many of the $\frac{1}{6}$ tiles exactly cover the $\frac{1}{2}$ tile?______

(b) Draw a sketch to show your result.

(c) Since ______ of the $\frac{1}{6}$ tiles cover the $\frac{1}{2}$ tile,

we see $\frac{\square}{6}$ is the same as $\frac{1}{2}$. $\qquad \frac{\square}{6} = \frac{1}{2}$

3) How many eighths equal one-half?______ $\qquad \frac{\square}{8} = \frac{1}{2}$

Draw a figure that demonstrates your answer.

4) How many tenths equal one-half?____ $\frac{\square}{10} = \frac{1}{2}$

Draw a figure that demonstrates your answer.

5) How many twelfths equal one-half?____ $\frac{\square}{12} = \frac{1}{2}$

Draw a figure that demonstrates your answer

6) Suppose you had bars marked — .

How many of them would it take to equal one-half?_____ $\frac{\square}{20} = \frac{1}{2}$

Take one of the $\frac{1}{3}$ bars and set it on your workspace.

7) How many sixths equal one-third?_____ $\frac{\square}{6} = \frac{1}{3}$

Draw a figure that demonstrates your answer.

8) How many twelfths equal one-third?_____ $\frac{\square}{12} = \frac{1}{3}$

Draw a figure that demonstrates your answer.

9) Suppose you had tiles marked $\frac{1}{30}$.

How many of them would it take to equal one-third? _____ $\frac{\square}{30} = \frac{1}{3}$

10) How many sixths equal two-thirds? _____ $\frac{\square}{6} = \frac{2}{3}$

Draw a figure that demonstrates your answer.

11) How many eighths equal three-fourths? _____ $\frac{\square}{8} = \frac{3}{4}$

Draw a figure that demonstrates your answer.

12) How many twelfths equal three-fourths? _____ $\frac{\square}{12} = \frac{3}{4}$

Draw a figure that demonstrates your answer.

13) Suppose you had tiles marked $\frac{1}{30}$.

(a) How many of them would it take to equal seven-tenths? _____ $\frac{\square}{30} = \frac{7}{10}$

(b) Explain how you got your answer.

14) Can you use twelfths to make a fraction equivalent to three-fifths? _______
Explain your reasoning.

Manipulative Mathematics **Name____________________**

Equivalent Fractions – Extra Practice

Use fraction tiles to do these exercises. You may want to use virtual fraction tiles on the interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html

1) How many eighths equal one-fourth? _____ $\frac{\square}{8} = \frac{1}{4}$

Draw a figure that demonstrates your answer.

2) How many twelfths equal one-third? _____ $\frac{\square}{12} = \frac{1}{3}$

Draw a figure that demonstrates your answer.

3) How many tenths equal four-fifths? _____ $\frac{\square}{10} = \frac{4}{5}$

Draw a figure that demonstrates your answer.

4) How many sixteenths equal three-fourths? _____ $\frac{\square}{16} = \frac{3}{4}$

Draw a figure that demonstrates your answer.

5) How many fifteenths equal two-thirds? _____ $\frac{\square}{15} = \frac{2}{3}$

Draw a figure that demonstrates your answer.

6) How many fifteenths equal two-fifths? _____ $\frac{\square}{15} = \frac{2}{5}$

Draw a figure that demonstrates your answer.

7) How many twelfths equal six-eighths? _____ $\frac{\square}{12} = \frac{6}{8}$

Draw a figure that demonstrates your answer.

8) How many twelfths equal six-ninths? _____ $\frac{\square}{12} = \frac{6}{9}$

Draw a figure that demonstrates your answer.

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Multiplying, Dividing, Adding, and Subtracting Fractions

Model Fraction Multiplication
Model Fraction Division
Model Fraction Addition
Model Fraction Subtraction
Model Finding the Least Common Denominator

Manipulatives used:

Fraction circles
Fraction tiles

Teacher video: Teaching Fraction Operations

Manipulative Mathematics **Instructor Page**

Model Fraction Multiplication

Resources Needed:

Each student needs the worksheet and two highlighters or colored pencils.

Background Information:

Many students who take this course have never been comfortable with fractions. They may have never made the connection between a concrete model of a fraction as part of a whole and the abstract concept and symbols. Fraction operations are merely rules that make no sense and thus are easily confused. By modeling the operations on fractions, students begin to understand how and why the procedures work, and may apply them more consistently. Working with models makes many students feel more competent and in control since it helps the procedures make sense.

The model of fraction multiplication used in this activity translates the expression $\frac{1}{2} \cdot \frac{3}{4}$ as "one-half of three-fourths". This is consistent with how fraction multiplication is applied in many real-life situations, such as two people sharing three-fourths of a pizza.

Directions:

- You may wish to review the definition of a fraction as part of a whole with your students before starting this activity. Understanding that concept is a prerequisite for this activity.
- This activity may be done by students in small groups or as individuals.
- Give each student the worksheet. Each student will need to use two highlighters or colored pencils.
- Demonstrate the first example of modeling multiplication, $\frac{1}{2} \cdot \frac{3}{4}$. Have students work along with you to complete parts (a) through (e).
- Let the class proceed through the worksheet activities. You may want to make sure everyone is actually modeling the multiplications rather than just writing the answers, as this is important to developing conceptual understanding.
- Discussion at the end of this activity will help reinforce the concepts. Make sure everyone can use the definition of Fraction Multiplication given in Exercise 5.
- The rectangle model of fraction multiplication can be found at the website: http://nlvm.usu.edu/en/nav/frames_asid_194_g_2_t_1.html?from=search.html?qt=multiply+fractions.

Manipulative Mathematics **Name___________________**

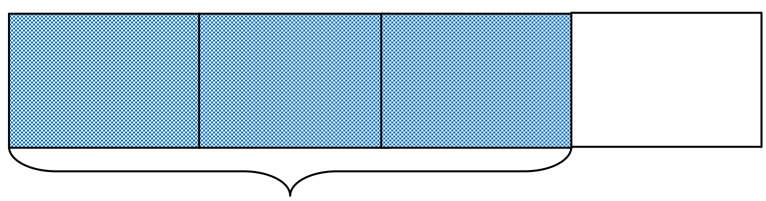
Model Fraction Multiplication

When you multiply fractions, do you need a common denominator? Do you take the reciprocal of one of the fractions? What are you supposed to do and how are you going to remember it? A model may help you understand multiplication of fractions.

1) Model the product $\frac{1}{2} \cdot \frac{3}{4}$.

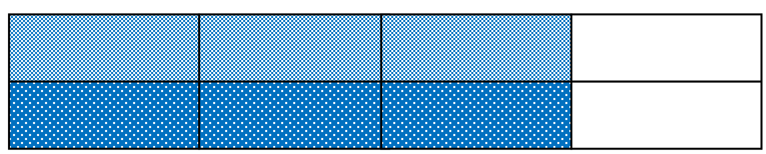
(a) To multiply $\frac{1}{2}$ and $\frac{3}{4}$, let's think "$\frac{1}{2}$ of $\frac{3}{4}$".

(b) First, we draw a rectangle to represent one whole. We divide it vertically into 4 equal parts, and then shade in three of the parts to model $\frac{3}{4}$.



We have shaded in $\frac{3}{4}$ of the rectangle.

(c) Now, we divide the rectangle horizontally into two equal parts to divide the whole into halves. Then we double-shade $\frac{1}{2}$ of what was already shaded.



(d) Into how many equal pieces is the rectangle divided now? ____________

(e) How many of these pieces are double-shaded? ________________

We double-shaded 3 out of the 8 equal pieces, $\frac{3}{8}$ of the rectangle. So $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$.

We showed that

$$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

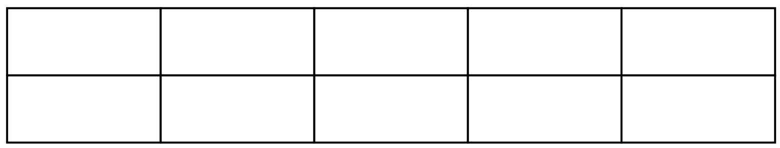
Notice –
multiplying the numerators $1 \cdot 3 = 3$
multiplying the denominators $2 \cdot 4 = 8$

2) Model the product $\frac{1}{2}\bullet\frac{3}{5}$.

(a) To multiply $\frac{1}{2}$ and $\frac{3}{5}$, think "$\frac{1}{2}$ of________".

(b) First shade in $\frac{3}{5}$ of the rectangle.

(c) Now double-shade $\frac{1}{2}$ of what was already shaded.



(d) Into how many equal pieces is the rectangle divided now? ___________

(e) How many pieces are double-shaded? ______________

(f) What fraction of the rectangle is double-shaded? _____________________

(g) So $\frac{1}{2}$ of $\frac{3}{5}$ is _______.

You have shown that

$$\frac{1}{2}\bullet\frac{3}{5}=\frac{3}{10}$$

Notice –
multiplying the numerators $1\bullet3=3$
multiplying the denominators $2\bullet5=10$

3) Use a rectangle to model each product. Sketch a diagram to illustrate your model.

(a) $\frac{1}{2}\bullet\frac{1}{3}$

$\frac{1}{2}\bullet\frac{1}{3}=$ ____

(b) $\frac{1}{2} \bullet \frac{1}{4}$

$\frac{1}{2} \bullet \frac{1}{4} =$ ____

(c) $\frac{1}{3} \bullet \frac{1}{4}$

$\frac{1}{3} \cdot \frac{1}{4} =$ ___

(d) $\frac{1}{3} \bullet \frac{2}{3}$

$\frac{1}{3} \bullet \frac{2}{3} =$ ___

(e) $\frac{2}{3} \bullet \frac{4}{5}$

$\frac{2}{3} \bullet \frac{4}{5} =$ ___

4) Look at each of your models and answers in Question 3.

(a) If you multiply numerators and multiply denominators, do you get the same result as you did from the model? _______

(b) Explain in words how to multiply two fractions.

5) The definition of fraction multiplication is given in the box below.

> **Fraction Multiplication**
>
> If a, b, c, and d are numbers where $b \neq 0$ *and* $d \neq 0$, then $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$.

To multiply fractions, multiply the numerators and multiply the denominators.

Use the definition of fraction multiplication to multiply $\frac{5}{12} \bullet \frac{7}{3}$

(a) Identify a, b, c, and d.

(b) Multiply the fractions.

Manipulative Mathematics **Name**______________________

Model Fraction Multiplication – Extra Practice

Use a rectangle to model each multiplication. Sketch your model and write the product.

You can practice using rectangles to model fraction multiplication online at the website: http://nlvm.usu.edu/en/nav/frames_asid_194_g_2_t_1.html?from=search.html?qt=multiply+fractions.

1) $\frac{1}{2} \bullet \frac{1}{6}$

2) $\frac{1}{2} \bullet \frac{1}{8}$

3) $\frac{1}{3} \bullet \frac{1}{3}$

4) $\frac{1}{4} \bullet \frac{1}{4}$

5) $\frac{1}{2} \bullet \frac{5}{8}$

6) $\frac{1}{2} \bullet \frac{5}{6}$

Multiply.

7) $\frac{2}{3} \bullet \frac{2}{5}$

8) $\frac{2}{5} \bullet \frac{4}{5}$

9) $\frac{3}{5} \bullet \frac{7}{8}$

10) $\frac{3}{4} \bullet \frac{5}{8}$

Model Fraction Division

Resources Needed:

Each student needs the worksheet and a set of fractions tiles.

Background Information:

Many students who take this course have never been comfortable with fractions. They may have never made the connection between a concrete model of a fraction as part of a whole and the abstract concept and symbols. Fraction operations are merely rules that make no sense and thus are easily confused. By modeling the operations on fractions, students begin to understand how and why the procedures work, and may apply them more consistently. Working with models makes many students feel more competent and in control since it helps make sense of the procedures.

The division model used in this activity is the 'goes into' model—for example, how many times does one-sixths go into one-half. This translates easily into 'how many one-sixths are there in one-half'. This model is consistent with division of whole numbers, a more familiar concept to most students, and may be presented as an extension of that idea.

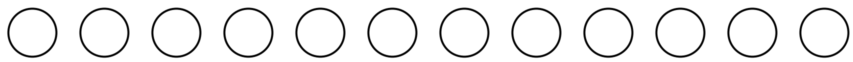
Directions:

- This activity may be done by students in small groups or as individuals.
- Give each student the worksheet and a set of fraction tiles.
- Model questions 1 and 2 from the worksheet together with the class. You may also want to talk about money to make fraction division real – how many quarters are in a half-dollar?
- Let the class proceed through the rest of the worksheet activities. You may want to make sure everyone is modeling the divisions rather than just writing the answers, as this is important to developing conceptual understanding.
- Discussion at the end of this activity will help reinforce the concepts. Question 8 is the only division that doesn't come out to be a whole number—this can lead to a good discussion of the size of the two fraction relative to each other. You may also want students to share their responses to Exercises 9 and 10.
- You might find it helpful to show your students the interactive fraction tile graphic at http://www.mathsisfun.com/numbers/fraction-number-line.html. You can move the slider and count, for example, how many twelfths are in three-fourths.

Model Fraction Division

1) Why is $12 \div 3 = 4$? Let's model this with counters.

   (a) How many groups of 3 counters can be made from the 12 shown below?



   (b) Draw a circle around each group of 3 counters. How many groups of 3 counters do you have? ____________

   (c) There are _____ groups of 3 counters. In other words, there are _____3s in 12. So, $12 \div 3 =$ _____ .

What about dividing fractions? Get out your fraction tiles and let's see!

2) To model the quotient $\frac{1}{2} \div \frac{1}{6}$ with fraction tiles we want to see how many sixths there are in one-half.

   (a) Line up your half and sixth fraction tiles as shown below.

<table>
<tr><td colspan="3" align="center">$\frac{1}{2}$</td></tr>
<tr><td align="center">$\frac{1}{6}$</td><td align="center">$\frac{1}{6}$</td><td align="center">$\frac{1}{6}$</td></tr>
</table>

   (b) How many $\frac{1}{6}$ s are in $\frac{1}{2}$? ________

   (c) $\frac{1}{2} \div \frac{1}{6} =$ ______

3) Model the quotient $\frac{1}{4} \div \frac{1}{8}$ with fraction tiles.

   Use your fourth and eighth fraction tiles to find out how many eighths there are in one fourth.

   (a) Draw a sketch of your result here.

(b) There are ______ $\frac{1}{8}$ s in $\frac{1}{4}$.

(c) So $\frac{1}{4} \div \frac{1}{8} =$ ______

4) Model the quotient $\frac{1}{3} \div \frac{1}{6}$ with fraction tiles

Use your third and sixth fraction tiles to find out how many sixths there are in one third

a) Draw a sketch of your result here.

b) There are ______ $\frac{1}{6}$ s in $\frac{1}{3}$.

c) So $\frac{1}{3} \div \frac{1}{6} =$ ______

5) Model the quotient $\frac{1}{2} \div \frac{1}{8}$ with fraction tiles

Use your half and eighth fraction tiles to find out how many eighths there are in one half.

a) Draw a sketch of your result here.

b) There are ______ $\frac{1}{8}$ s in $\frac{1}{2}$

c) So $\frac{1}{2} \div \frac{1}{8} =$ ______

Model a Whole Number Divided by a Fraction

6) Use fraction bars to model the quotient $2 \div \frac{1}{4}$

(a) How many $\frac{1}{4}$ s are there in 2?

1				1			
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(b) There are ______ $\frac{1}{4}$ s in 2, so $2 \div \frac{1}{4} =$ _____

(c) Let's think of this example another way—in terms of money. We often read $\frac{1}{4}$ as 'one quarter', so you can think of $2 \div \frac{1}{4}$, as asking "how many quarters are there in two dollars?" We know that $1 is 4 quarters, so how many quarters are in $2? _______

(d) So, $2 \div \frac{1}{4} =$ _______ .

7) Use fraction tiles to model the following. Sketch a diagram to illustrate your model.

a) $2 \div \frac{1}{3}$

b) $3 \div \frac{1}{2}$

$2 \div \frac{1}{3} =$ _____

$3 \div \frac{1}{2} =$ _____

Using fraction tiles in exercise 2, we showed that $\frac{1}{2} \div \frac{1}{6} = 3$. Notice that $\frac{1}{2} \bullet \frac{6}{1} = 3$ also.

How does $\frac{6}{1}$ relate to $\frac{1}{6}$? They are reciprocals! To divide fractions, we multiply the first fraction by the reciprocal of the second. This leads to the following definition.

Fraction Division

If $a, b, c,$ and d are numbers where $b \neq 0, c \neq 0$ and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$

8) Use the Fraction Division definition above to find the quotient $\frac{5}{7} \div \frac{3}{8}$.

(a) Identify the numbers that correspond to $a, b, c,$ and d .

(b) Divide the fractions.

9) Explain in words how to divide two fractions.

10) Explain in words how to divide a whole number by a fraction.

 Name____________________

Model Fraction Division – Extra Practice

Use fraction tiles to model each division. Sketch your model and write the quotient.

You may want to use the fraction tiles shown at the interactive website:
http://www.mathsisfun.com/numbers/fraction-number-line.html.

1) $\frac{1}{2} \div \frac{1}{10}$

2) $\frac{1}{2} \div \frac{1}{12}$

3) $\frac{1}{3} \div \frac{1}{12}$

4) $\frac{1}{4} \div \frac{1}{12}$

5) $\frac{3}{4} \div \frac{1}{8}$

6) $\frac{2}{5} \div \frac{1}{10}$

Divide.

7) $\frac{5}{8} \div \frac{1}{6}$

8) $\frac{5}{6} \div \frac{1}{8}$

9) $\frac{2}{5} \div \frac{1}{2}$

10) $\frac{3}{10} \div \frac{1}{3}$

Model Fraction Addition

Resources Needed:

Each student needs the worksheet and a set of fractions circles.

Background:

Many students who take this course have never been comfortable with fractions. Fraction addition and subtraction are especially problematic. Students don't understand why adding or subtracting across numerators and/or denominators is incorrect. Working with concrete models helps students see why a common denominator is needed.

Directions:

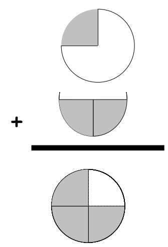
- This activity may be done by students individually or in small groups. The addition and subtraction worksheets may be done at separate times.
- Give each student a set of fraction circles and the worksheet.
- Demonstrate the example of modeling $\frac{1}{4}+\frac{2}{4}$ using quarters. Many students do not think of quarters as fractions of dollars. It is important that they see that relationship and are able to use to it as a reference point for fraction addition.
- Then use fraction circles to model the same addition, $\frac{1}{4}+\frac{2}{4}$.
- Let the class proceed through the worksheet activities.
- When most of the students have completed the worksheet, bring the whole class together. Discussion at the end of this activity will help reinforce the concepts. Since the goal is to have students understand addition of fractions with common denominators, you may want to accept answers that are not simplified, such as $\frac{6}{8}$.
- You may want to show your students how to use the fraction circles on the website: http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

Manipulative Mathematics **Name___________________**

Model Fraction Addition

How many quarters are pictured above? One quarter plus 2 quarters equals 3 quarters. Quarters? Remember, quarters are really fractions of a dollar; “quarter” is another word for “fourth”. So the picture of the coins shows that $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$.

Let’s use fraction circles to model addition of fractions for the same example, $\frac{1}{4}+\frac{2}{4}$.



Start with one $\frac{1}{4}$ piece. $\frac{1}{4}$

Add two more $\frac{1}{4}$ pieces. $+\frac{2}{4}$

The result is $\frac{3}{4}$. $\frac{3}{4}$

So, $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$.

1) Use fraction circles to model the sum $\frac{3}{8}+\frac{2}{8}$.

(a) Take three $\frac{1}{8}$ pieces. Add two more $\frac{1}{8}$ pieces. How many $\frac{1}{8}$ pieces do you have? ___

(b) Sketch your model here.

(c) You have five eighths. $\frac{3}{8}+\frac{2}{8}=$ _____

2) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{1}{3}+\frac{1}{3}=$ _____ .

(b) $\frac{1}{6}+\frac{4}{6}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

3) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{1}{5}+\frac{3}{5}=$ _____

(b) $\frac{2}{5}+\frac{2}{5}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

4) Use fraction circles to model the following. Sketch a diagram to illustrate your model**.**

(a) $\frac{3}{8}+\frac{4}{8}=$ _____

(b) $\frac{1}{8}+\frac{4}{8}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

5) A common error made by students when adding fractions is to add the numerators and add the denominators (much like we multiply numerators and multiply denominators when multiplying fractions). Use a model to see why this does not work for addition!

(a) Model $\frac{1}{5}+\frac{1}{5}$. Sketch a diagram to illustrate your model.

(b) Did the fifths change to another size piece?_____ Did they change to $\frac{1}{10}$ pieces?_____

(c) $\frac{1}{5}+\frac{1}{5}=$ _____

These examples show that to add the same size fraction pieces—that is, fractions with the same denominator—you just add the number of pieces. So, to add fractions with the same denominator, you add the numerators and place the sum over the common denominator. This leads to the following definition.

Fraction Addition

If $a, b, and\ c$ are numbers where $c \neq 0$, then $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

6) Use the definition of fraction addition in the box above to add $\frac{6}{23} + \frac{8}{23}$.

(c) Identify a, b, and c .

(d) Add the fractions.

.

7) Explain in words how to add two fractions that have the same denominator.

Manipulative Mathematics **Name__________________**

Model Fraction Addition – Extra Practice

Use fraction circles to model each addition. Sketch your model and write the sum.

You may want to use the fraction circles on the interactive website: http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

1) $\frac{1}{5}+\frac{2}{5}$

2) $\frac{1}{6}+\frac{2}{6}$

3) $\frac{3}{8}+\frac{1}{8}$

4) $\frac{4}{10}+\frac{1}{10}$

5) $\frac{3}{10}+\frac{3}{10}$

6) $\frac{5}{12}+\frac{5}{12}$

7) $\frac{5}{9}+\frac{3}{9}$

8) $\frac{3}{8}+\frac{4}{8}$

9) $\frac{4}{5}+\frac{2}{5}$

10) $\frac{4}{9}+\frac{6}{9}$

11) $\frac{5}{8}+\frac{7}{8}$

12) $\frac{7}{10}+\frac{9}{10}$

Model Fraction Subtraction

Resources Needed:

Each student needs the worksheet and a set of fractions circles.

Background:

Many students who take this course have never been comfortable with fractions. Fraction addition and subtraction are especially problematic. Students don't understand why adding or subtracting across numerators and/or denominators is incorrect. Working with concrete models helps students see why a common denominator is needed.

Directions:

- This activity may be done by students individually or in small groups. The addition and subtraction worksheets may be done at separate times.
- Give each student a set of fraction circles and the worksheet.
- Discuss the example of modeling subtraction involving pizza. You may want to illustrate this on the board.
- Then use fraction circles to model the same subtraction, $\frac{7}{12}-\frac{2}{12}$.
- Let the class proceed through the worksheet activity.
- When most of the students have completed the worksheet, bring the whole class together. Discussion at the end of this activity will help reinforce the concepts. Since the goal is to have students understand subtraction of fractions with common denominators, you may want to accept answers that are not simplified, such as $\frac{6}{8}$.
- You may want to show your students how to use the fraction circles on the website: http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

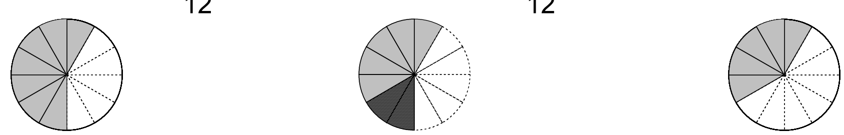
Manipulative Mathematics **Name___________________**

Model Fraction Subtraction

Subtracting two fractions with common denominators works the same as addition of fractions with common denominators. Think of a pizza that was cut into twelve equal slices. Each piece is $\frac{1}{12}$ of the pizza. After dinner there are seven pieces, $\frac{7}{12}$, left in the box. If Leonardo eats 2 of the pieces, $\frac{2}{12}$, how much is left? There would be 5 pieces left, $\frac{5}{12}$. So $\frac{7}{12}-\frac{2}{12}=\frac{5}{12}$.

1) Let's use Fraction Circles to model the same example, $\frac{7}{12}-\frac{2}{12}$.

(a) Start with seven $\frac{1}{12}$ pieces. Take away two $\frac{1}{12}$ pieces.



How many twelfths do you have left?_____

(b) You have five pieces left, $\frac{5}{12}$.

$\frac{7}{12}-\frac{2}{12}=$ _____

2) Use your fraction circles to model the difference $\frac{4}{5}-\frac{1}{5}$.

Start with four $\frac{1}{5}$ pieces. Take away one $\frac{1}{5}$ piece.

(a) How many fifths do you have left?_____

(b) Sketch your model here.

(c) You have ________ fifths left.

$\frac{4}{5}-\frac{1}{5}=$ _____

3) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{7}{8}-\frac{4}{8}=$ _____

(b) $\frac{5}{6}-\frac{4}{6}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

4) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{3}{4}-\frac{2}{4}=$ _____ (b) $\frac{4}{5}-\frac{2}{5}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

5) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{5}{8}-\frac{2}{8}=$ _____ (b) $\frac{7}{10}-\frac{4}{10}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

These examples show that to subtract the same size fraction pieces—that is, fractions with the same denominator—you just subtract the number of pieces. So, to subtract fractions with the same denominator, you subtract the numerators and place the difference over the common denominator. This leads to the following definition.

> **Fraction Subtraction**
>
> If $a, b, and\ c$ are numbers where $c \neq 0$, then $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$

6) Use the definition of fraction subtraction in the box above to subtract $\frac{11}{17}-\frac{5}{17}$.

(a) Identify $a, b,$ and c .

(b) Subtract the fractions.

7) Explain in words how to subtract two fractions that have the same denominator.

Manipulative Mathematics **Name___________________**

Model Fraction Subtraction – Extra Practice

Use fraction circles to model each subtraction. Sketch your model and write the difference.

You may want to use the fraction circles on the interactive website: http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

1) $\frac{3}{5}-\frac{1}{5}$

2) $\frac{5}{6}-\frac{1}{6}$

3) $\frac{7}{8}-\frac{1}{8}$

4) $\frac{9}{10}-\frac{1}{10}$

5) $\frac{3}{10}-\frac{3}{10}$

6) $\frac{5}{12}-\frac{5}{12}$

7) $\frac{5}{9}-\frac{3}{9}$

8) $\frac{4}{8}-\frac{3}{8}$

9) $\frac{6}{5}-\frac{2}{5}$

10) $\frac{10}{9}-\frac{4}{9}$

11) $\frac{13}{8}-\frac{5}{8}$

12) $\frac{17}{10}-\frac{7}{10}$

Model Finding the Least Common Denominator

Resources Needed:

Each student needs a worksheet and a set of fractions tiles or fraction circles.

Background:

Many students who take this course have never been comfortable with fractions. Addition and subtraction of fractions with different denominators is confusing and often meaningless. While they may remember when a common denominator is needed, they generally have no conceptual understanding of what that common denominator means. As a result, when asked to convert to equivalent fractions with an LCD they follow routine procedures without a visual image of what they are really accomplishing.

Directions:

- This activity may be done by students in small groups or as individuals. Each student should complete his or her own worksheet.
- Give each student a set of fraction tiles or fraction circles and a worksheet. Since the activity can be done using either fraction tiles or fraction circles, the worksheet refers generically to 'fraction pieces'.
- Talk through the example of adding one quarter and one dime. Most students will say they 'just know' the sum is 35 cents. Explicitly relate this to the idea of a common denominator. You might want to remind them that the coin 'quarter' is named because its value is one-quarter $\left(\frac{1}{4}\right)$ of one dollar.
- Then model exercise 1, parts (a) through (d). Make sure students understand what is meant when they are asked 'to find a common fraction piece that can be used to cover both $\frac{1}{2}$ and $\frac{1}{3}$ exactly'. Show them how they can cover $\frac{1}{2}$ exactly with $\frac{1}{4}$ pieces but they cannot cover $\frac{1}{3}$ exactly with $\frac{1}{4}$ pieces. (Let your students know whether you prefer them to answer question 1(d) by saying 'it takes two $\frac{1}{4}$ pieces to cover $\frac{1}{3}$' or 'it cannot be done'.)
- Let the class proceed through the worksheet activities. Walk around the room, to make sure all students are on task and to answer any individual questions that arise.
- Discussion at the end of this activity will help reinforce the concepts. The worksheet does not use the phrase 'equivalent fraction', but students do convert fractions to equivalent fractions with the LCD, so you may want to point that out.
- You may want to show your students how to use the fraction tiles on the website http://www.mathsisfun.com/numbers/fraction-number-line.html or the fraction circles at http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

Model Finding the Least Common Denominator

Let's look at coins again. Can you add one quarter and one dime? Well, you could say there are two coins, but that's not very useful. To find the total value of one quarter plus one dime, you change them to the same kind of unit – cents. One quarter equals 25 cents and one dime equals 10 cents, so the sum is 35 cents.

One quarter + one dime

25¢ + 10¢

35¢

25¢ + 10¢

$\frac{25}{100} + \frac{10}{100}$

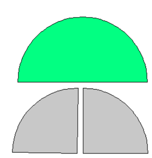
$\frac{35}{100}$

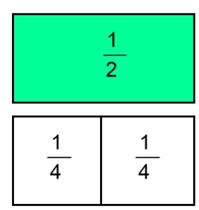
Similarly, when you add fractions with different denominators you have to convert them to equivalent fractions with a common denominator. With the coins, when we converted to cents, the denominator was 100. 25 cents is $\frac{25}{100}$ and 10 cents is $\frac{10}{100}$ and so we added $\frac{25}{100}+\frac{10}{100}$ to get $\frac{35}{100}$, which is 35 cents.

- Use fraction pieces to find the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$. Take out your set of fraction pieces and place $\frac{1}{2}$ and $\frac{1}{3}$ on your workspace. You need to find a common fraction piece that can be used to cover both $\frac{1}{2}$ and $\frac{1}{3}$ **exactly**.

1) Try the $\frac{1}{4}$ pieces.

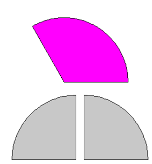
fraction circles fraction tiles

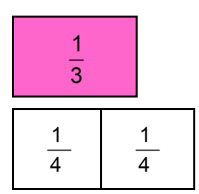




(a) Can you cover the $\frac{1}{2}$ piece exactly with $\frac{1}{4}$ pieces? _____

(b) How many $\frac{1}{4}$ pieces cover the $\frac{1}{2}$ piece? _____





(c) Can you cover the $\frac{1}{3}$ piece exactly with $\frac{1}{4}$ pieces? _____

(d) How many $\frac{1}{4}$ pieces cover the $\frac{1}{3}$ piece?_____

(e) Sketch your results here.

2) Try the $\frac{1}{5}$ pieces.

(a) Can you cover the $\frac{1}{2}$ piece exactly with $\frac{1}{5}$ pieces? _____

(b) How many $\frac{1}{5}$ pieces cover the $\frac{1}{2}$ piece?___

(c) Can you cover the $\frac{1}{3}$ piece exactly with $\frac{1}{5}$ pieces?_____.

(d) How many $\frac{1}{5}$ pieces cover the $\frac{1}{3}$ piece?___

(e) Sketch your results here.

3) Try the $\frac{1}{6}$ pieces.

(a) Can you cover the $\frac{1}{2}$ piece exactly with $\frac{1}{6}$ pieces? _____

(b) How many $\frac{1}{6}$ pieces cover the $\frac{1}{2}$ piece? ___

(c) Can you cover the $\frac{1}{3}$ piece exactly with $\frac{1}{6}$ pieces? _____.

(d) How many $\frac{1}{6}$ pieces cover the $\frac{1}{3}$ piece? ___

(e) Sketch your results here.

4) You have shown that:

(a) 3 of the $\frac{1}{6}$ pieces exactly cover the $\frac{1}{2}$ piece. $\quad \frac{1}{2} = \frac{}{6}$

(b) 2 of the $\frac{1}{6}$ pieces exactly cover the $\frac{1}{3}$ piece. $\quad \frac{1}{3} = \frac{}{6}$

The smallest denominator of a fraction piece that can be used to cover both fractions exactly is the **least common denominator** (LCD) of the two fractions. The smallest denominator of a fraction piece that can be used to cover both $\frac{1}{2}$ and $\frac{1}{3}$ is 6. So, you have found that the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$ is 6.

- Use fraction pieces to find the least common denominator of $\frac{1}{4}$ and $\frac{1}{6}$. Place $\frac{1}{4}$ and $\frac{1}{6}$ on your workspace. Find a common fraction piece that can be used to cover both $\frac{1}{4}$ and $\frac{1}{6}$ exactly.

5) Sketch your results here.

6) You have shown that:

(a) _____ of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{4}$ piece. $\frac{1}{4} = \frac{}{}$

(b) _____of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{6}$ piece. $\frac{1}{6} = \frac{}{}$

(c) Both fractions can be written with denominator_____, so _____ is their common denominator.

- Use fraction pieces to find the least common denominator of $\frac{1}{4}$ and $\frac{1}{3}$. Find a common fraction piece that can be used to cover both $\frac{1}{4}$ and $\frac{1}{3}$ exactly.

7) Sketch your results here.

8) You have shown that:

(a) _____ of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{4}$ piece . $\frac{1}{4} = \frac{}{}$

(b) _____of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{3}$ piece. $\frac{1}{3} = \frac{}{}$

(c) Both fractions can be written with denominator_____, so ____ is their common denominator.

- Use fraction pieces to find the least common denominator of $\frac{1}{2}$ and $\frac{1}{5}$. Find a common fraction piece that can be used to cover both $\frac{1}{2}$ and $\frac{1}{5}$ exactly.

  9) Sketch your results here.

  10) You have shown that:

  (a) _____ of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{2}$ piece. $\frac{1}{2} = \frac{}{}$

  (b) _____of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{5}$ piece. $\frac{1}{5} = \frac{}{}$

  (c) Both fractions can be written with denominator_____, so ____ is their common denominator.

Manipulative Mathematics **Name___________________**

Model Finding the Least Common Denominator – Extra Practice

Use fraction tiles or fraction circles to find the least common denominator (LCD) of each pair of fractions, and to re-write each fraction with the LCD. Sketch your model.

You may want to use the fraction tiles on the interactive website: http://www.mathsisfun.com/numbers/fraction-number-line.html or the fraction circles at http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html to work these exercises.

1) $\frac{1}{3}$ and $\frac{1}{6}$

(a) LCD = _______

(b) $\frac{1}{3}$ = _________

(c) $\frac{1}{6}$ = _________

(d) sketch your model.

2) $\frac{1}{2}$ and $\frac{1}{8}$

(a) LCD = _______

(b) $\frac{1}{2}$ = _________

(c) $\frac{1}{8}$ = _________

(d) sketch your model.

3) $\frac{2}{3}$ and $\frac{1}{12}$

(a) LCD = _______

(b) $\frac{2}{3}$ = _________

(c) $\frac{1}{12}$ = _________

(d) sketch your model.

4) $\frac{3}{4}$ and $\frac{1}{12}$

(a) LCD = _______

(b) $\frac{3}{4}$ = _________

(c) $\frac{1}{12}$ = _________

(d) sketch your model.

5) $\frac{3}{8}$ and $\frac{3}{4}$

(a) LCD = _______

(b) $\frac{3}{8}$ = _________

(c) $\frac{3}{4}$ = _________

(d) sketch your model.

6) $\frac{5}{6}$ and $\frac{2}{3}$

(a) LCD = _______

(b) $\frac{5}{6}$ = _________

(c) $\frac{1}{2}$ = _________

(d) sketch your model.

7) $\frac{1}{2}$ and $\frac{2}{5}$

(a) LCD = ________

(b) $\frac{1}{2}$ = __________

(c) $\frac{2}{5}$ = __________

(d) sketch your model.

8) $\frac{1}{3}$ and $\frac{3}{4}$

(a) LCD = ________

(b) $\frac{1}{3}$ = __________

(c) $\frac{3}{4}$ = __________

(d) sketch your model.

9) $\frac{5}{12}$ and $\frac{2}{3}$

(a) LCD = ________

(b) $\frac{5}{12}$ = __________

(c) $\frac{2}{3}$ = __________

(d) sketch your model.

10) $\frac{3}{4}$ and $\frac{5}{6}$

(a) LCD = ________

(b) $\frac{3}{4}$ = __________

(c) $\frac{5}{6}$ = __________

(d) sketch your model.

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Signed Numbers

Addition of Signed Numbers
Subtraction of Signed Numbers

Manipulatives used:
Two color counters

Teacher video: Teaching Signed Number Operations

Addition of Signed Numbers

Resources Needed:

Each student needs about 30 two-color counters. If you have no supply money to purchase counters, buy a bag of red beans and a bag of white beans at a grocery store. Beans are inexpensive and easy to replace, so students can take them home and use them to do the homework. Each student needs about 15 red beans and 15 white beans.

Background Information:

Most students understand addition and subtraction of positive numbers since they have worked with them for many years, but when negative numbers are introduced students encounter difficulties. Students tend to merely want a 'rule' to follow just to get the answer. By using two-color counters, students have a concrete model of the abstract concepts of signed numbers, and develop the 'rules' themselves. Furthermore, the model for subtraction of signed numbers agrees with the 'take away' idea most students used when they first learned subtraction as children.

Directions:

- This activity introduces addition of signed numbers.
- Give each student about 30 two-color counters (or about 15 red beans and 15 white beans).
- State that the red side of a counter (or one red bean) represents one negative unit and the other side (one white bean) represents one positive unit. Using red for negative is consistent with the 'red ink' used in accounting.
- Explain that one positive and one negative together make a 'neutral pair'. The value of a neutral pair is zero.
- Demonstrate worksheet exercises 1 through 4, have the students model with their counters, too. When you get a neutral pair, physically remove it from the workspace.
- After working exercises 1 through 4 together, you may want your students to model $6+4$, $6+(-4)$, $-6+4$, $-6+(-4)$ on their own and then reinforce the correct methods by modeling them yourself for the class.
- Let students work in groups of 2 or 3 on the rest of the worksheet. Make sure everyone is actually modeling the sums instead of just writing the answers. It will be important that they can model sums before attempting subtraction.
- When most groups are finished, bring the class back together. Discuss their answers to exercises 17 and 18.
- Ideally, addition would be introduced in class one day and the students would do addition exercises for homework. Then subtraction would be introduced at the next class meeting.
- Students can get additional practice using two color counters to add signed numbers online at the National Library of Virtual Manipulatives website: http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html?from=topic_t_1.html.

Manipulative Mathematics **Name**______________________

Addition of Signed Numbers

Team
Members____

We are going to model signed numbers with two-color counters. One white counter, ○, will represent one positive unit. One red counter, ●, will represent one negative unit.

When we have one positive and one negative together, ○ ● we call it a 'neutral pair'. The value of a neutral pair is zero.

1) We'll start by modeling $5+3$, the sum of 5 and 3.
   (a) Start with 5 positives. ○ ○ ○ ○ ○

   (b) Add 3 positives. Put counters of the same color in the same row. ○ ○ ○ ○ ○ ○ ○ ○

   (c) How many counters are there? __________ positives

   $5+3=8$

2) Now we'll model $-5+(-3)$, the sum of negative 5 and negative 3.
   (a) Start with 5 negatives. ● ● ● ● ●

   (b) Add 3 negatives. ● ● ● ● ● ● ● ●

   (c) How many counters are there? __________ negatives

   $-5+(-3)=-8$

3) What about adding numbers with different signs? Let's model $-5+3$, the sum of negative 5 and 3.
   (a) Start with 5 negatives. ● ● ● ● ●

   (b) Add 3 positives. Since they are a different color, line them up under the red counters.
   ● ● ● ● ●
   ○ ○ ○

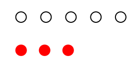
   (c) Are there any neutral pairs? _______
   Remove the neutral pairs.
   ● ● ● ● ●
   ○ ○ ○

   (d) How many left? ● ● ________ negatives

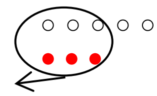
   $-5+3=-2$

4) The fourth case is the sum of a positive and a negative. We'll model $5+(-3)$, the sum of 5 and negative 3.
   (a) Start with 5 positives. ○ ○ ○ ○ ○

(b) Add 3 negatives.



(c) Remove the neutral pairs.



(d) How many are left?

○ ○ 2 ______________

$5+(-3)=2$

Use your counters to model each sum. Draw a sketch of your model.

5) $4+2$

6) $-5+(-5)$

7) $-1+4$

8) $2+(-4)$

9) $8+(-4)$

10) $7+(-3)$

11) $-2+(-3)$

12) $-5+7$

13) $-2+(-1)$

14) $-3+3$

15) $7+(-2)$

16) $-4+2$

17) Do you notice a pattern? Explain in words how to add:

(a) $-8+(-10)$

(b) $25+(-5)$

18) Without using counters, try to find these sums.

(a) $35+29$

(b) $-57+(-43)$

(c) $78+(-74)$

(d) $-64+31$

Manipulative Mathematics **Name____________________**

Addition of Signed Numbers – Extra Practice

Use two-color counters to model each addition.

You can find virtual counters on the website: http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html?from=topic_t_1.html. If you use the website, click on 'User' at the bottom of the workspace so that you can enter the numbers in each exercise.

Sketch the model for each addition and find the sum.

1) $5+2$ 2) $-5+(-2)$ 3) $-5+2$ 4) $5+(-2)$

5) $6+(-6)$ 6) $3+(-1)$ 7) $-4+(-5)$ 8) $-6+8$

9) $-3+(-7)$ 10) $-2+2$ 11) $4+(-8)$ 12) $-4+9$

Subtraction of Signed Numbers

Resources Needed:

Each student needs about 30 two-color counters. If you have no supply money to purchase counters, buy a bag of red beans and a bag of white beans at a grocery store. Beans are inexpensive and easy to replace, so students can take them home and use them to do the homework. Each student needs about 15 red beans and 15 white beans.

Background Information:

Most students understand addition and subtraction of positive numbers since they have worked with them for many years, but when negative numbers are introduced students encounter difficulties. Students tend to merely want a 'rule' to follow just to get the answer. By using two-color counters, students have a concrete model of the abstract concepts of signed numbers, and develop the 'rules' themselves. Furthermore, the model for subtraction of signed numbers agrees with the 'take away' idea most students used when they first learned subtraction as children.

Directions:

- This activity introduces subtraction of signed numbers. Students will discover that $a-b=a+(-b)$. Ideally, students would have been introduced to addition of signed numbers at the previous class meeting and have done homework to practice.
- Give each student about 30 two-color counters (or about 15 red beans and 15 white beans).
- Remind students that the red side of a counter (or one red bean) represents one negative unit and the other side (one white bean) represents one positive unit. Review the idea of a 'neutral pair' – one positive and one negative together. The value of a neutral pair is zero.
- Demonstrate worksheet exercises 1 through 4; have the students model with their counters, too. When you get a neutral pair, physically remove it from the workspace.
- After working exercises 1 through 4 together, you may want your students to model $6-4$, $-6-(-4)$, $-6-4$, $6-(-4)$ on their own and then reinforce the correct methods by modeling them yourself for the class.
- Let students work in groups of 2 or 3 on the rest of the worksheet. Make sure everyone is actually modeling the differences instead of just writing the answers. A quick look at the answers to exercises 13 and 14 will let you know if the students understand the process.
- When most groups are finished, bring the class back together. Discuss their answers to exercises 17 and 18.
- Students can get additional practice using two color counters to subtract signed numbers online at the National Library of Virtual Manipulatives website: http://nlvm.usu.edu/en/nav/frames_asid_162_g_3_t_1.html?from=topic_t_1.html.

Manipulative Mathematics **Name**______________________

Subtraction of Signed Numbers

Team Members____

We are going to model signed numbers with two-color counters. One white counter, ○, will represent one positive unit. One red counter, ●, will represent one negative unit.

When we have one positive and one negative together, ○ ● we call it a 'neutral pair'. The value of a neutral pair is zero.

1) We'll start by modeling $5-3$, the difference of 5 and 3.
   (a) Start with 5 positives. ○ ○ ○ ○ ○
   (b) Take away 3 positives. ○ ○ ○ ○ ○
   (c) How many counters are left? ________ positives

   $5-3=2$

2) Now we'll model $-5-(-3)$, the difference of negative 5 and negative 3.
   (a) Start with 5 negatives. ● ● ● ● ●
   (b) Take away 3 negatives. ● ● ● ● ●
   (c) How many counters are left? ● ● ________ negatives

   $-5-(-3)=-2$

3) What about subtracting numbers with different signs? Let's model $-5-3$, the difference of negative 5 and 3.
   (a) Start with 5 negatives. ● ● ● ● ●
   (b) We want to take away 3 positives. Do we have any positives to take away? __________
   (c) We can add 3 neutral pairs to get the 3 positives. ● ● ● ● ● ● ● ● / ○ ○ ○
   (d) Now take away 3 positives. ● ● ● ● ● ● ● ● / ○ ○ ○
   (e) How many counters are left? ● ● ● ● ● ● ● ●

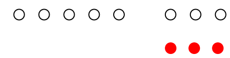
   8 ____________

   $-5-3=-8$

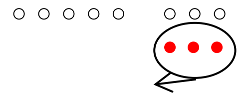
4) The fourth case is the sum of a positive and a negative. We'll model $5-(-3)$, the difference of 5 and negative 3.
   (a) Start with 5 positives. ○ ○ ○ ○ ○

(b) We want to take away 3 negatives. Do we have any negatives to take away? ____________

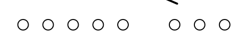
(c) But we can add 3 neutral pairs to get the 3 negatives.



(d) Take away 3 negatives.



(e) How many counters are left?



8 ______________

$5-(-3)=8$

Use your counters to model each difference. Draw a sketch of your model.

5) $7-2$

6) $6-(-4)$

7) $-1-4$

8) $-3-(-2)$

9) $3-(-4)$

10) $-5-(-1)$

11) $8-6$

12) $-7-3$

13) $-4-(-4)$

14) $-3-3$

15) $1-5$

16) $-2-(-6)$

17) Do you notice a pattern? Explain in words how to subtract:

(a) $-8-(-2)$

(b) $-10-5$

18) Without using counters, try to find these differences.

(a) $35-29$

(b) $-57-(-43)$

(c) $78-(-74)$

(d) $-64-31$

 Name________________________

Subtraction of Signed Numbers – Extra Practice

Use two-color counters to model each subtraction.

You can find virtual counters on the website: http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html?from=topic_t_1.html. If you use the website, click on 'User' at the bottom of the workspace so that you can enter the numbers in each exercise.

Sketch the model for each subtraction and find the difference.

1) $7-2$

2) $7-(-2)$

3) $-7-2$

4) $-7-(-2)$

5) $6-(-5)$

6) $-4-(-1)$

7) $-8-8$

8) $9-5$

9) $-3-(-3)$

10) $5-4$

11) $-2-(-6)$

12) $4-10$

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Multiples and Primes

Multiples
Prime Numbers

Manipulatives used:
Hundreds Charts

Teacher video: [Teaching Concepts of Multiples and Primes]

Multiples

Resources Needed:

Each student needs the worksheets and a highlighter pen.

Background Information:

Students often rely on rote memorization of definitions and have little understanding of the concept of a multiple of a number. They often confuse multiples with factors. This hands-on activity reinforces the definition of multiple and helps students develop the skills needed in future work with multiples in topics such as least common multiple and lowest common denominator.

Directions:

- This activity may be done by individual students or in a small group.
- Give each student the worksheets and ask them to take out a highlighter pen.
- Introduce the definition of a multiple of a number. You may wish to note that when counting by 5's, for example, you are listing multiples of 5. Then help students start highlighting the multiples of 2. You may want to project a copy of the number chart and show the class how to begin highlighting multiples of 2 in the first row.
- Let students proceed through the worksheet on their own or in groups. Circulate around the class to make sure everyone is on task and to offer clarification when needed.
- When most students have finished the worksheet, bring the class together for discussion. Solicit input from the students about the patterns they discovered and the rules they created. Test their rules on some numbers other than those on the worksheet.
- There is a good online hundreds chart at http://nlvm.usu.edu/en/nav/frames_asid_158_g_3_t_1.html?open=instructions&from=topic_t_1.html . Click 'Show Multiples' to highlight the multiples of each number you choose.

Manipulative Mathematics **Name**_____________________

Multiples

Multiple of a Number
A number is a multiple of n if it is the product of a counting number and n.

1) **Multiples of 2**

(a) This table lists the counting numbers from 1 to 50. Highlight all the multiples of 2.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

(b) Now look at all the numbers that you highlighted. Describe a pattern you notice.

(c) Create a rule you could use to determine if a number larger than 50 is a multiple of 2.

(d) Use your rule to decide if 497 is a multiple of 2.

(e) Is 846 a multiple of 2?

2) **Multiples of 5**

(a) This table lists the counting numbers from 1 to 50. Highlight all the multiples of 5.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

(b) Now look at all the numbers that you highlighted. Describe a pattern you notice.

(c) Create a rule you could use to determine if a number larger than 50 is a multiple of 5.

(d) Use your rule to decide if 741 is a multiple of 5.

(e) Is 940 a multiple of 5?

3) **Multiples of 10**

(a) The table lists the counting numbers from 1 to 50. Highlight all the multiples of 10.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

(b) Now look at all the numbers that you highlighted. Describe a pattern you notice.

(c) Create a rule you could use to determine if a number larger than 50 is a multiple of 10.

(d) Use your rule to decide if 690 is a multiple of 10.

(e) Is 875 a multiple of 10?

4) **Multiples of 3**

(a) The table lists the counting numbers from 1 to 50. Highlight all the multiples of 3.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

(b) List the multiples of 3.

(c) Under each multiple of 3, find the sum of the digits of that number. For example, 42 is a multiple of 3, and $4+2=6$. What do you notice about all the multiples of 3?

	3,	6,	9,	12,	15,	18,..........,	42,...
sum of digits	3	6	9	$\underbrace{1+2}_{3}$	$\underbrace{1+5}_{6}$	$\underbrace{1+8}_{9}$	$\underbrace{4+2}_{6}$

(d) Use these results to create a rule to determine if a number is a multiple of 3.

(e) Use your rule to decide if 375 is a multiple of 3.

(f) Is 1488 a multiple of 3?

Manipulative Mathematics **Name______________________**

Multiples – Extra Practice

1) State a rule you can use to determine if a number is a multiple of:

(a) 2 ____

(b) 3 ____

(c) 5 ____

(d) 10 ____

For each number, determine if it is a multiple of 2, 3, 5, and/or 10, and indicate your answers by writing 'yes' or 'no' in the spaces below.

		(a) multiple of 2	(b) multiple of 3	(c) multiple of 5	(d) multiple of 10
2)	130	(a)_________	(b) _________	(c) _________	(d) _________
3)	165	(a) _________	(b) _________	(c) _________	(d) _________
4)	225	(a) _________	(b) _________	(c) _________	(d) _________
5)	234	(a) _________	(b) _________	(c) _________	(d) _________
6)	255	(a) _________	(b) _________	(c) _________	(d) _________
7)	270	(a) _________	(b) _________	(c) _________	(d) _________
8)	625	(a) _________	(b) _________	(c) _________	(d) _________
9)	1155	(a) _________	(b) _________	(c) _________	(d) _________
10)	1650	(a) _________	(b) _________	(c) _________	(d) ________

Prime Numbers

Resources Needed:

Each student needs the worksheets and a highlighter pen.

Background Information:

Students often rely on rote memorization of definitions and have little understanding of the concept of prime numbers. This hands-on activity reinforces the definition of prime numbers as students cross out multiples of numbers on a hundreds chart, and eventually are left with only primes. (This is the 'Sieve of Eratosthenes'.) The activity demonstrates that prime numbers have no factors other than themselves and one. It helps students develop a clearer understanding of prime numbers and a confidence in their own ability to recognize whether a number is prime.

Directions:

- This activity may be done by individual students or in a small group.
- Give each student the worksheets and ask them to take out a highlighter pen.
- Review the definitions of prime and composite numbers. To illustrate the definitions, give an example of a prime number and of a composite number.
- Help the class get started. You might show the class the number chart and begin by drawing a circle around 2 and then crossing out the multiples of 2 in the first row. Then repeat this process for 3, so students get the idea.
- Let students proceed through the worksheet on their own or in groups. Circulate around the class to make sure everyone is on task and to offer clarification when needed.
- Class discussion afterward will help reinforce the concepts. List all the primes less than 50, and have the students talk about their answers to the last question.
- An online version of the the Sieve of Eratosthenes activity can be found at http://nlvm.usu.edu/en/nav/frames_asid_158_g_3_t_1.html?open=instructions&from=topic_t_1.html. Click 'Remove Multiples' at the bottom of the workspace, and then you'll end up with only the primes showing.

Manipulative Mathematics **Name______________________**

Prime Numbers

Prime Number
A prime number is a counting number greater than 1, whose only factors are one and itself.
A counting number that is not prime is **composite**.

1) Use this table to find the primes less than 50. Remember a prime number is a number whose only factors are 1 and itself. The number 1 is not considered prime, so the smallest prime number is 2.

   (a) On the table, circle 2 and then cross out all the multiples of 2. All multiples of 2, greater than 2, have two as a factor and so are not prime.

   (b) Next, circle 3 and then cross out all the multiples of 3. All multiples of 3, greater than 3, have three as a factor and so are not prime.

   (c) Go to the next number that has not been crossed out. Circle it—it is prime—and then cross out all its multiples.

   (d) Continue this routine until all the numbers in the table have been crossed out or circled.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

2) The numbers that have been crossed out are not prime. Counting numbers that are not prime are called ______________________________.

3) The circled numbers are prime. List the primes less than 50.

4) What are the only factors of each prime you listed?

5) State one fact you notice about the primes.

Manipulative Mathematics **Name________________________**

Prime Numbers – Extra Practice

1) Use this table to find the primes less than 100. Remember a prime number is a number whose only factors are 1 and itself. The number 1 is not considered prime, so the smallest prime number is 2.

   (a) On the table, circle 2 and then cross out all the multiples of 2. All multiples of 2, greater than 2, have two as a factor and so are not prime.

   (b) Next, circle 3 and then cross out all the multiples of 3. All multiples of 3, greater than 3, have three as a factor and so are not prime.

   (c) Go to the next number that has not been crossed out. Circle it—-it is prime—and then cross out all its multiples.

   (d) Continue this routine until all the numbers in the table have been crossed out or circled.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2) The circled numbers are prime. List the primes less than 100.

For more practice online, you can use the hundreds chart at http://nlvm.usu.edu/en/nav/frames_asid_158_g_3_t_1.html?open=instructions&hidepanel=true&from=topic_t_1.html. Display 10 rows to show the numbers 1 to 100 and click 'Remove Multiples' at the bottom of the workspace to remove (instead of crossing out) the multiples of each prime.

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Solving Equations

Subtraction Property of Equality

Division Property of Equality

Manipulatives used:

Small envelopes

Counters

Teacher video: Teaching Equations

Subtraction Property of Equality

Resources Needed:

Each student needs the worksheet, a small envelope, and about 20 counters.

Background Information

Students often think of solving equations as merely a mechanical process where you "move" numbers and variables across an equal sign. The "rules" don't make sense and so students often take steps that, even if they are algebraically correct, don't bring the equation closer to a solution. This exercise gives students a concrete model and guides their thought processes. By using the concrete model to develop the Subtraction Property of Equality, students are able to make the logical progression to the Addition Property of Equality.

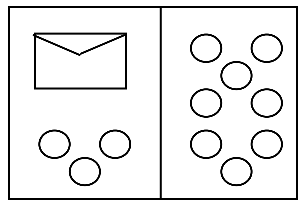
Directions:

- This activity is best done with students in pairs.
- Do the first example together, using a projector, if possible. Set it up beforehand by putting 5 counters in an envelope. Then arrange the envelope and counters as shown in the figure. As you lead the class through it, make sure the students actually move the counters—this is important for the brain. When you have isolated the envelope, dump the counters out of it, to show that it does, indeed, contain 5 counters.
- Usually, students readily accept the natural progression from the model to the equation.
- Let the students continue on the worksheet. Some classes may not need to do all the exercises on the worksheet, but make sure all pairs of students do the last two exercises where they create equations for each other.
- Class discussion afterward will help reinforce the concepts. You may want to ask students to solve a few simple equations using the Subtraction Property of Equality with larger numbers.

Manipulative Mathematics **Name____________________**

Subtraction Property of Equality

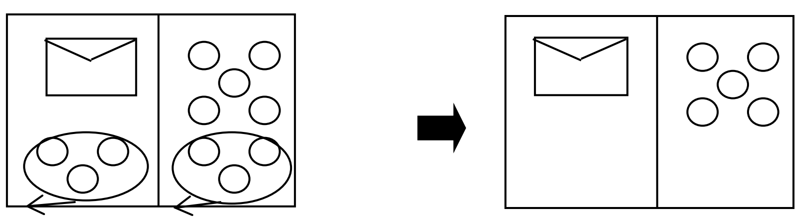
1) You are going to solve a puzzle. Use your envelopes and counters to recreate the picture below on your workspace. Both sides have the same number of counters, but some counters are “hidden” in the envelope. The goal is to discover how many counters are in the envelope.



(a) How many counters are in the envelope? ______counters are in the envelope.

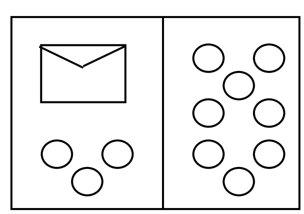
(b) What are you thinking? What steps are you taking in your mind to figure out how many counters are in the envelope? List the steps here.

Perhaps you are thinking- the 3 counters at the bottom left can be matched with 3 on the right. Then I can take them away from both sides. That leaves five on the right-so there must be 5 counters in the envelope. Try this with your envelope and counters.



(c) Each side of the workspace models an expression and the line in the middle represents the equal sign, so we can write an algebraic equation from this model.

What algebraic equation is modeled by this picture?



______________ = ________

Let's write algebraically the steps we took to discover how many counters were in the envelope:

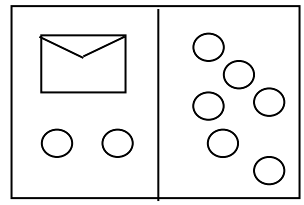
$$x + 3 = 8$$

We took away three from each side. $\quad x + 3 - \_\_\_ = 8 - \_\_\_$

And then we had ____ left. $\quad x = 5$

(d) Check: $\_\_\_ + 3 = 8$
Five in the envelope plus three more equals eight!

2) Let's try this again! How many counters are in the envelope?



Use your envelope and counters to recreate this picture. Now, move the counters to find out how many counters are in the envelope.

(a) List the steps you took to find out how many counters were in the envelope.

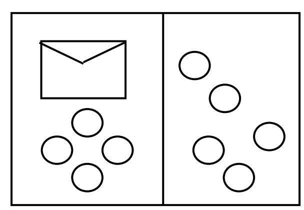
(b) What algebraic equation is modeled by this picture? $\quad x + \_\_\_\_\_ = \_\_\_\_\_$

(c) We need to take away ______from each side. $\quad x + 2 - \_\_\_\_ = 6 - \_\_\_\_$

(d) There are _____counters in the envelope! $\quad x = \_\_\_\_$

(e) Check: $\_\_\_ + 2 = 6$
Four in the envelope plus two more does equal six!

3) How many counters are in this envelope?



Use your envelope and counters to recreate this picture. Move the counters to discover how many counters are in the envelope.

(a) Write the algebraic equation that is modeled by this picture. $\quad x + \_\_\_\_\_ = \_\_\_\_\_$

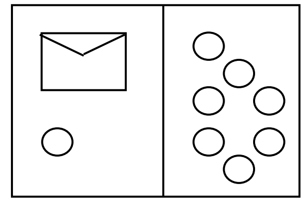
(b) Take away _____ from each side. $\quad x + 4 - \_\_\_\_ = 5 - \_\_\_\_$

(c) There are _____counters in the envelope! $\quad x = \_\_\_\_$

(d) Check: $\_\_\_ + 4 = 5$

4) How many counters are in this envelope?

Use your envelope and counters to recreate this picture. Move the counters to find the number of counters in the envelope.



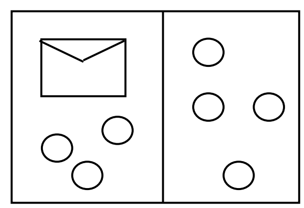
(a) Write the equation modeled by the envelope and counters. __________=___________

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

5) How many counters are in this envelope?

Use your envelopes and counters to recreate this picture. Move the counters as needed to find the number of counters in the envelope.



(a) Write the equation modeled by the envelope and counters. __________=___________

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

6) Model a similar equation for your partner. Have your partner figure out how many counters are in the envelope.
   (a) Sketch a picture of your model.

   (b) Show the algebra steps your partner took to find the number of counters in the envelope.

7) Have your partner model a similar equation for you. Figure out how many counters are in the envelope.
   (a) Sketch a picture of the model.

   (b) Show the algebra steps you took to find the number of counters in the envelope.

With these puzzles we have modeled a method for solving one kind of equation. To solve each equation, we used the Subtraction Property of Equality.

> **The Subtraction Property of Equality:**
> For any real numbers a, b, and c,
> if $a = b$, then $a - c = b - c$.

When you subtract the same quantity from both sides of an equation, you still have equality!

Manipulative Mathematics **Name_____________________**

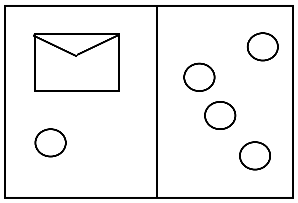
Subtraction Property of Equality – Extra Practice

#1-6: For each figure:

(a) Write the equation modeled by the envelopes and counters.

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

1)

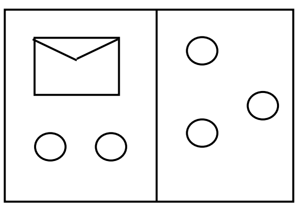


(a) Equation ___________=__________

(b) Solution

Words	Algebra

2)

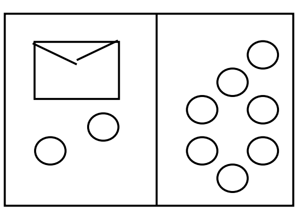


(a) Equation ___________=__________

(b) Solution

Words	Algebra

3)

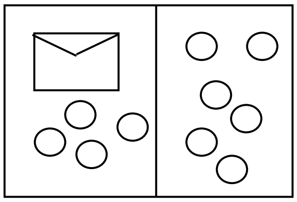


(a) Equation ___________=__________

(b) Solution

Words	Algebra

4)

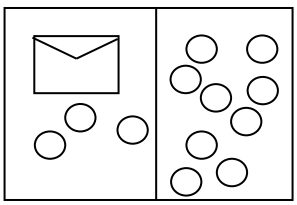


(b) Solution

Words	Algebra

(a) Equation ____________=___________

5)

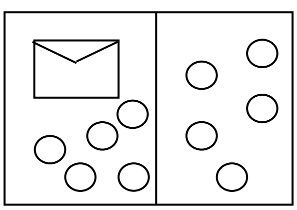


(b) Solution

Words	Algebra

(a) Equation ____________=___________

6)



(b) Solution

Words	Algebra

(a) Equation ____________=___________

#7-18: Solve each equation using the Subtraction Property of Equality.

7)
$$x+3=5$$
$$x+3-\_\_\_=5-\_\_\_$$
$$x=\_\_\_$$

8)
$$x+2=10$$
$$x+2-\_\_\_=10-\_\_\_$$
$$x=\_\_\_$$

9)
$$x+9=17$$
$$x+9-\_\_\_=17-\_\_\_$$
$$x=\_\_\_$$

10)
$$x+14=23$$
$$x+14-\_\_\_=23-\_\_\_$$
$$x=\_\_\_$$

11)
$$x+36=51$$
$$x+36-\_\_\_=51-\_\_\_$$
$$x=\_\_\_$$

12)
$$x+75=102$$
$$x+75-\_\_\_=102-\_\_\_$$
$$x=\_\_\_$$

13) $x+18=33$

14) $y+29=100$

15) $u+72=241$

16) $v+325=465$

17) $m+593=902$

18) $n+762=2014$

Division Property of Equality

Resources Needed:

Each student needs the worksheet, a few small envelopes and about 20 counters.

Background Information

Students often think of solving equations as merely a mechanical process in which you "move" numbers and variables across an equal sign. The "rules" don't make sense and so students often take steps that, even if they are algebraically correct, don't bring the equation closer to a solution. This exercise gives students a concrete model and guides their thought processes. By using the concrete model to develop the Division Property of Equality, students are able to make the logical progression to the Multiplication Property of Equality.

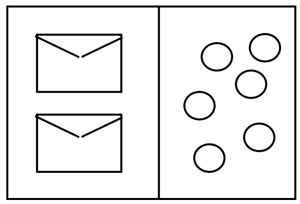
Directions:

- This activity is best done with students in pairs.
- Do the first example together, using a projector, if possible. Set it up beforehand with 2 envelopes containing 3 counters in each. Then arrange the envelope and counters as shown in the figure. As you lead the class through it, make sure the students actually move the counters—this is important for the brain. When you have separated the envelopes, dump the counters out of each one, to show that they do, indeed, each contain 3 counters.
- Usually, students follow the natural progression to the equation very well.
- Let the students continue on the worksheet. Some classes may not need to do all the exercises on the worksheet, but make sure all pairs of students do the last two exercises where they create equations for each other.
- Class discussion afterward will help reinforce the concepts. You may want to ask students to solve a few simple equations using the Division Property of Equality with bigger numbers.

Manipulative Mathematics **Name___________________**

Division Property of Equality

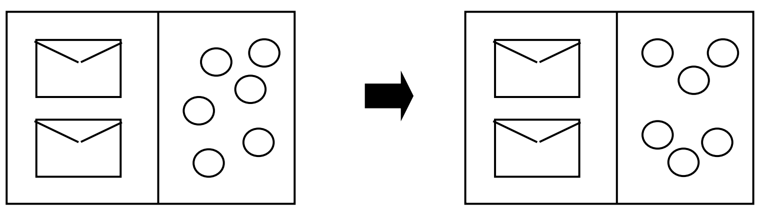
1) You are going to solve a puzzle. Use your envelopes and counters to recreate the picture below on your workspace. Both sides have the same total number of counters, but some counters are "hidden" in the envelopes. Both envelopes contain the same number of counters. The goal is to discover how many counters are in each envelope.



(a) How many counters are in each envelope? _____counters are in each envelope.

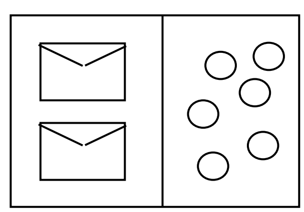
(b) What are you thinking? What steps are you taking in your mind to figure out how many counters are in each envelope? List the steps here.

Perhaps you are thinking that you have to separate the counters on the right side into 2 groups, because there are 2 envelopes. So 6 counters divided into 2 groups means there must be 3 counters in each envelope. Try this with your envelopes and counters.



(c) Each side of the workspace models an expression and the line in the middle represents the equal sign, so we can write an algebraic equation from this model.

What algebraic equation is modeled by this picture?



___________ = ______

(d) Let's write algebraically the steps we took to discover how many counters were in the envelope:

$$2x = 6$$

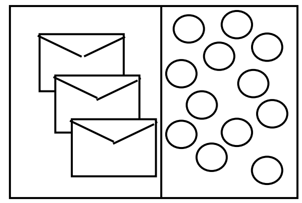
We divided both sides of the equation by _____, $\frac{2x}{\square} = \frac{6}{\square}$

So we have _____ in each envelope. $x = 3$

(e) Check: $2 \bullet \_\_\_ = 6$ Three counters in each of two envelopes equals six!

2) Here's another puzzle. How many counters are in each envelope?

Use your envelopes and counters to recreate this picture. Now, move the counters to find out how many counters are in each envelope.



(a) List the steps you took to find out how many counters are in each envelope.

(b) What algebraic equation is modeled by this picture? $\_\_\_ x = \_\_\_\_\_$

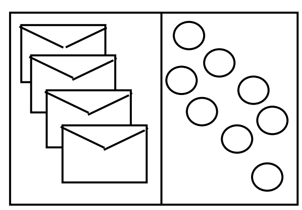
(c) We need to divide the counters into _____groups.

(d) Divide each side by _____. $\frac{3x}{\square} = \frac{12}{\square}$

(e) There are _____counters in each envelope! $x = \_\_\_\_$

(f) Check: $3 \bullet \_\_\_ = 12$

3) How many counters are in each envelope?
Use your envelopes and counters to recreate this picture. Move the counters to discover how many counters are in each envelope.



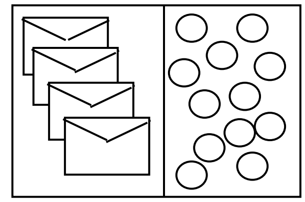
(a) Write the algebraic equation that would match this situation. $\_\_\_ x = \_\_\_\_\_$

(b) Divide each side by_______. $\frac{4x}{\square} = \frac{8}{\square}$

(c) There are _____counters in each envelope! $x = \_\_\_\_$

(d) Check: $4 \bullet \_\_\_ = 8$

4) How many counters are in each envelope?
Use your envelopes and counters to recreate this picture.
Move the counters to find the number of counters in the envelope.

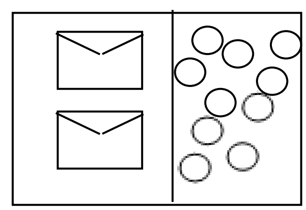


(a) Write the equation modeled by the envelopes and counters. $\_\_\_ x = \_\_\_\_\_$

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

5) How many counters are in each envelope?
Use your envelopes and counters to recreate this picture.
Move the counters as needed to find the number of counters in the envelope.



(a) Write the equation modeled by the envelopes and counters. $\_\_\_ x = \_\_\_\_\_$

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

6) Model a similar equation for your partner. Have your partner figure out how many counters are in each envelope.
   (a) Sketch a picture of your model.

   (b) Show the algebra steps your partner took to find the number of counters in each envelope.

7) Have your partner model a similar equation for you. Figure out how many counters are in each envelope.
   (a) Sketch a picture of the model.

   (b) Show the algebra steps you took to find the number of counters in each envelope.

With these puzzles we have modeled a method for solving one kind of equation. To solve each equation, we used the Division Property of Equality.

The Division Property of Equality

For any real numbers a, b, c, and $c \neq 0$,

if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

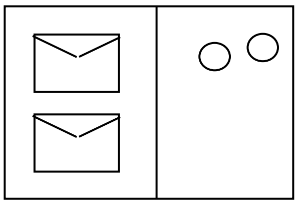
When you divide both sides of an equation by any non-zero number, you still have equality!

Manipulative Mathematics **Name**______________________

Division Property of Equality – Extra Practice

#1-6: For each figure:
(a) write the equation modeled by the envelopes and counters.
(b) show the steps you take, in words and algebra, to find the number of counters in each envelope.

1)

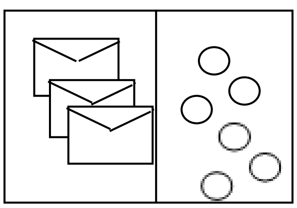


(a) Equation ___ $x =$ _____

(b) Solution

Words	Algebra

2)

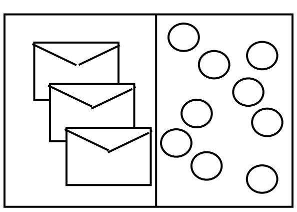


(a) Equation ___ $x =$ _____

(b) Solution

Words	Algebra

3)

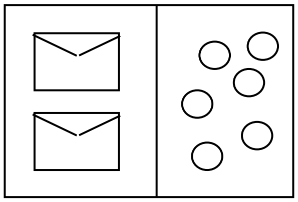


(a) Equation ___ $x =$ _____

(b) Solution

Words	Algebra

4)

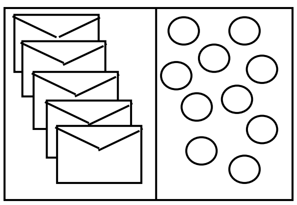


(a) Equation ___ $x =$ ______

(b) Solution

Words	Algebra

5)

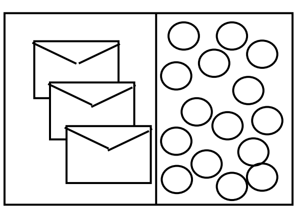


(a) Equation ___ $x =$ ______

(b) Solution

Words	Algebra

6)



(a) Equation ___ $x =$ ______

(b) Solution

Words	Algebra

#7-18: Solve each equation using the Division Property of Equality.

7)

$$2x = 16$$

$$\frac{2x}{\boxed{}} = \frac{16}{\boxed{}}$$

$x =$ ___

8)

$$4x = 16$$

$$\frac{4x}{\boxed{}} = \frac{16}{\boxed{}}$$

$x =$ ___

9)

$$8x = 16$$

$$\frac{8x}{\boxed{}} = \frac{16}{\boxed{}}$$

$x =$ ___

10)

$$5x = 35$$

$$\frac{5x}{\boxed{}} = \frac{35}{\boxed{}}$$

$x =$ ___

11)

$$9x = 54$$

$$\frac{9x}{\boxed{}} = \frac{54}{\boxed{}}$$

$x =$ ___

12)

$$12x = 108$$

$$\frac{12x}{\boxed{}} = \frac{108}{\boxed{}}$$

$x =$ ___

13) $7x = 42$

14) $11n = 165$

15) $19y = 38$

16) $25q = 375$

17) $80p = 800$

18) $101m = 909$

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Area and Perimeter

Visualizing Area and Perimeter
Measuring Area and Perimeter

Manipulatives used:
Square ‘color’ tiles

Teacher video: Teaching Area and Perimeter

Manipulative Mathematics **Instructor Page**

Visualizing Area and Perimeter

Resources Needed:

Each student needs 7 color tiles and a sheet of grid paper.

Background Information:

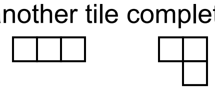
Linear and square measurements have little meaning for many students. This activity is designed to have students work with perimeter and area concretely and better understand the concepts of linear and square measurement. Students will also differentiate between perimeter and area.

Directions:

- Give each student a set of 7 color tiles and a sheet of grid paper.

- Use one tile to explain the concepts of linear measurement and perimeter, and square measurement and area. Show students:
  - The perimeter of the tile is 4 inches.
  - The area is one square inch.
  - This is the only shape possible with just 1 tile.

- Project 3 tiles and ask your students:
  - How many different shapes can be made with 3 tiles? Explain that to create a shape with more than one tile, all the tiles must be used and that each tile must touch another tile completely along one side. (So there are just 2 shapes made with 3 tiles.)



  - What is the perimeter of each shape?
  - What is the area of each shape?

- Have students do Exercises 1, 2, and 3 on the worksheet. Working in groups seems to work best.

- When most groups are finished with Exercise 3, making shapes from 4 tiles, you may want to discuss their results. Ask each group to show one of their shapes on the board, and to write the perimeter and area. (Perimeters of 8 and 10 are possible - any others?) Some groups may show the same shape, but with a different orientation, giving you an opportunity to talk about rotations and flips, etc. When all possible shapes of 4 tiles have been shown, have your students record them shapes on their grid paper.

- Then ask your students, still working in groups, to finish the worksheet where they will find various shapes using 5 tiles. Students may also investigate making shapes with 6 and/or 7 tiles if time permits.

- Bring the class together for a quick debriefing of their responses to Exercises 6 and 7.

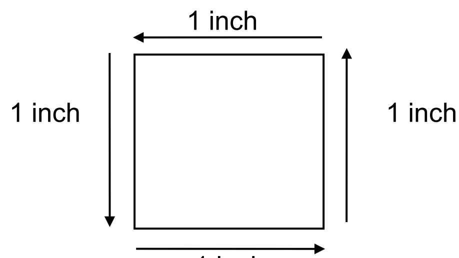
- You may want to use the square blocks at the interactive website http://nlvm.usu.edu/en/nav/frames_asid_169_g_1_t_3.html?open=activities&from=topic_t_3.html to show the different shapes you can make using 3 tiles.

Manipulative Mathematics **Name_______________________**

Visualizing Area and Perimeter

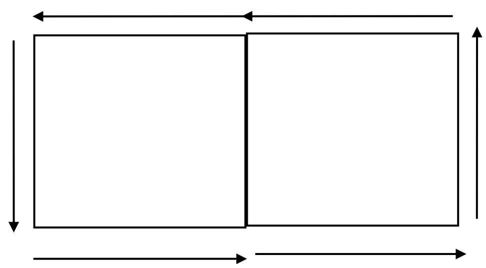
Team Members____

A color tile is a square that is 1 inch on a side. If an ant walked around the edge of the tile, it would have walked 4 inches. This distance around the tile is called the **perimeter** of the tile. The **area** of the tile is measured by determining how many square inches (or other unit) cover the tile. Since a color tile is a square that is 1 inch on each side, its area is one square inch.



Perimeter is 4 inches.
Area is 1 square inch.

1) Use 2 tiles to make a shape like the one shown below. Notice that each tile must touch the other along one complete side.



(a) What is the perimeter of this shape? Perimeter =_____

(b) What is the area? Area =____

(c) Can you make any other shape using two tiles? ____

(d) Can you find any other perimeter using two tiles? ____

(e) Record your results in the chart in #5.

2) Make all possible shapes with 3 tiles. Keep in mind that rotations and flips are really the same shape! Sketch your shapes on your grid paper, and color or shade in the squares.

(a) How many shapes did you make? ____

(b) For each shape, find its perimeter. Write the perimeter next to each shape.

(c) What is the area of each shape that you made? Write the area inside each shape.

(d) Record your results in the chart in #5.

3) Now use 4 tiles. Sketch all the possible shapes on your grid paper.
   (a) How many shapes did you make? ____
   (b) For each shape, find its perimeter. Write the perimeter next to each shape.
   (c) What is the area of each shape that you made? Write the area inside each shape.
   (d) Record your results in the chart in #5.

4) Take 5 tiles. Sketch all the possible shapes on your grid paper.
   (a) How many shapes did you make? ____
   (b) For each shape, find its perimeter. Write the perimeter next to each shape.
   (c) List all the perimeters of the shapes with 5 tiles.

   (d) Was more than one shape possible for any perimeter? ____
   (e) What is the smallest perimeter possible using 5 tiles? _____ Why?

   (f) What is the largest perimeter possible using 5 tiles? _____ Why?

   (g) What is the area of each shape that you made? Write the area inside each shape.
   (h) List all the areas of the shapes with 5 tiles.

   (i) Record your results in the chart in #5.

5) Fill in the chart below to show your results from #1-4.

Number of tiles	Perimeters Found	Areas Found
1	4 inches	1 square inch
2		
3		
4		
5		

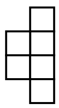
6) Name one fact you learned about perimeter from this activity.

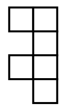
7) Name one fact you learned about area from this activity.

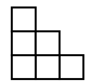
Manipulative Mathematics **Name________________________**

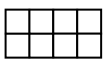
Visualizing Area and Perimeter – Extra Practice

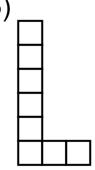
Find the area and perimeter of each shape.

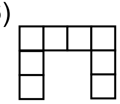
1)

area= ________
perimeter=____

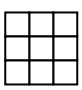
2)

area= ________
perimeter=____

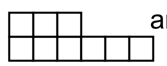
3)

area= ________
perimeter=____

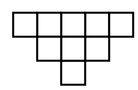
4)

area= ________
perimeter=____

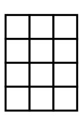
5)

area= ________
perimeter=____

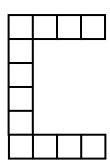
6)

area= ________
perimeter=____

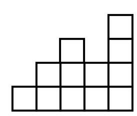
7)

area= ________
perimeter=____

8)

area= ________
perimeter=____

9)

area= ________
perimeter=____

10)

area= ________
perimeter=____

11)

area= ________
perimeter=____

12)

area= ________
perimeter=____

For more practice, use color tiles to make your own shapes and then find the area and perimeter. You may want to use the square blocks at the interactive website:
http://nlvm.usu.edu/en/nav/frames_asid_169_g_1_t_3.html?open=activities&from=topic_t_3.html

Manipulative Mathematics **Instructor Page**

Measuring Area and Perimeter

Resources Needed:

Each student needs a set of 20 color tiles and the worksheets, including Shapes I through VI. It may be helpful if you can project Shape I through use of an overhead projector or document camera.

Background Information:

Linear and square measurements have little meaning for many students. This activity is designed to have students work with area and perimeter concretely and better understand the concept of linear and square measurement.

Directions:

- This activity is best done individually. Give each student a set of 20 color tiles and worksheets.
- Explain the concepts of linear measurement and perimeter, and square measurement and area.
- Show the class Shape I.
- Have the students estimate how many tiles will cover the shape. Record this on the board and label it 'Estimated Area'. Be sure the estimate is in square inches. Then have the students estimate the perimeter of the shape, and label it 'Estimated Perimeter'. The units will be inches.
- Then cover the shape completely with tiles and count the number of tiles needed. Write that on the board labeled 'Measured Area', again using square inches. Now count the number of tiles along the perimeter of the shape, and label that 'Measured Perimeter'. Have your students briefly discuss how good they are at estimating area and perimeter.
- Direct your students to use their color tiles to complete the worksheet, estimating and measuring Shapes II through VI.
- When most students have finished, discuss their answers to Exercises 3 and 4 with the class.

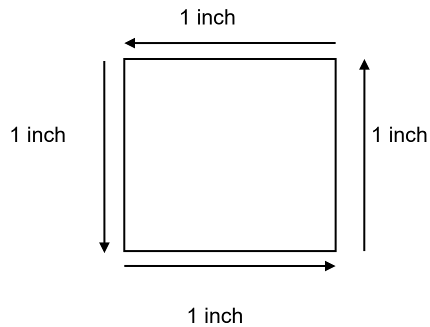
As an alternative to the using the shape pages, you might want to create your own shapes online using the geoboard activity at http://nlvm.usu.edu/en/nav/frames_asid_282_g_3_t_3.html?open=activities&from=topic_t_3.html. You can create a shape by stretching a rubber band around the pegs, and then click on 'Measures' to see the area and perimeter. To be consistent with the shapes on the worksheets, be sure to use only right angles.

Manipulative Mathematics **Name____________________**

Measuring Area and Perimeter

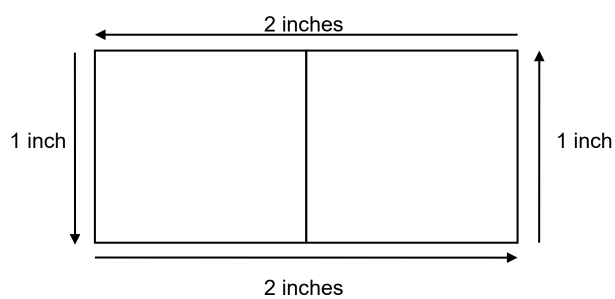
The **area** of a shape is measured by determining how many square inches (or other unit) cover the shape. The **perimeter** is the distance around the shape.

A color tile is a square that is 1 inch long on each side.
Its area is one square inch. Its perimeter is 4 inches.



Perimeter is 4 inches.
Area is 1 square inch.

If we put two tiles side by side we have a shape with area two square inches. The perimeter is 6 inches, because the distance along a side of each square is 1 inch.



Perimeter is 6 inches.
Area is 2 square inches.

1) Take your set of tiles and Shape I.

   (a) First, estimate how many tiles will be needed to completely cover Shape I. Record this in the 'Estimated Area' column on the chart below.

   (b) Next, estimate how many tiles will form the perimeter of Shape I. Record this in the 'Estimated Perimeter' column on the chart below.

   (c) Now cover Shape I completely with tiles. Count the number of tiles you used and record this in the 'Measured Area' column in the chart on the next page. Count the number of tiles along the perimeter and record this in the 'Measured Perimeter' column.

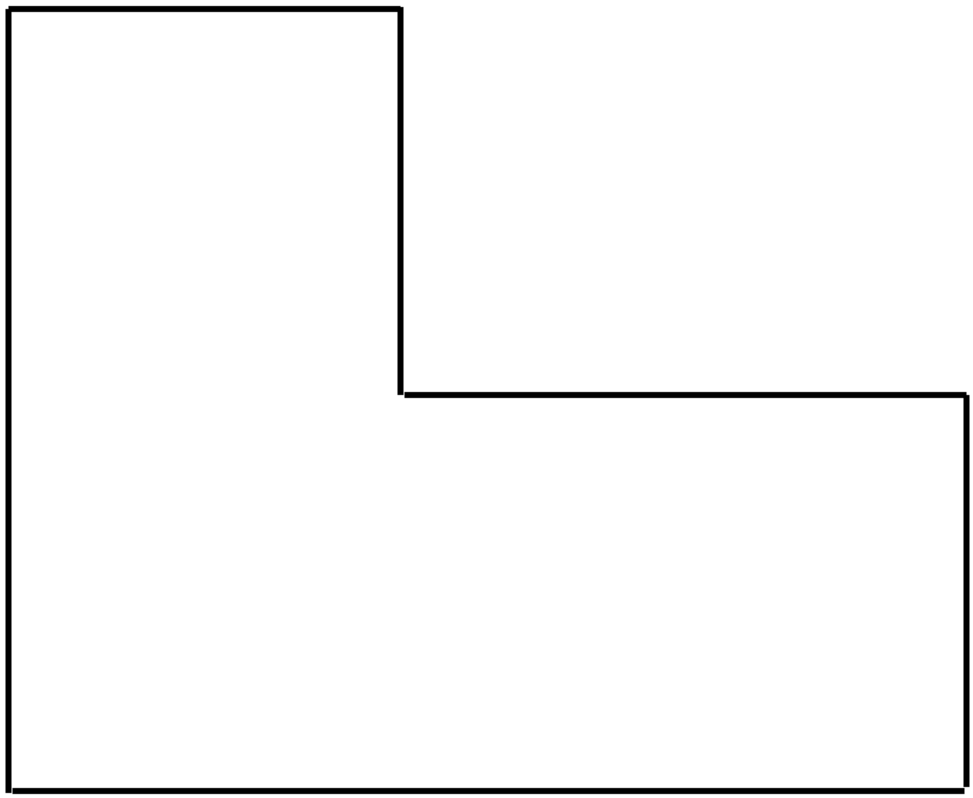
2) Repeat this process with the rest of your shapes.

Shape	Estimated Area	Estimated Perimeter	Measured Area	Measured Perimeter
I				
II				
III				
IV				
V				
VI				

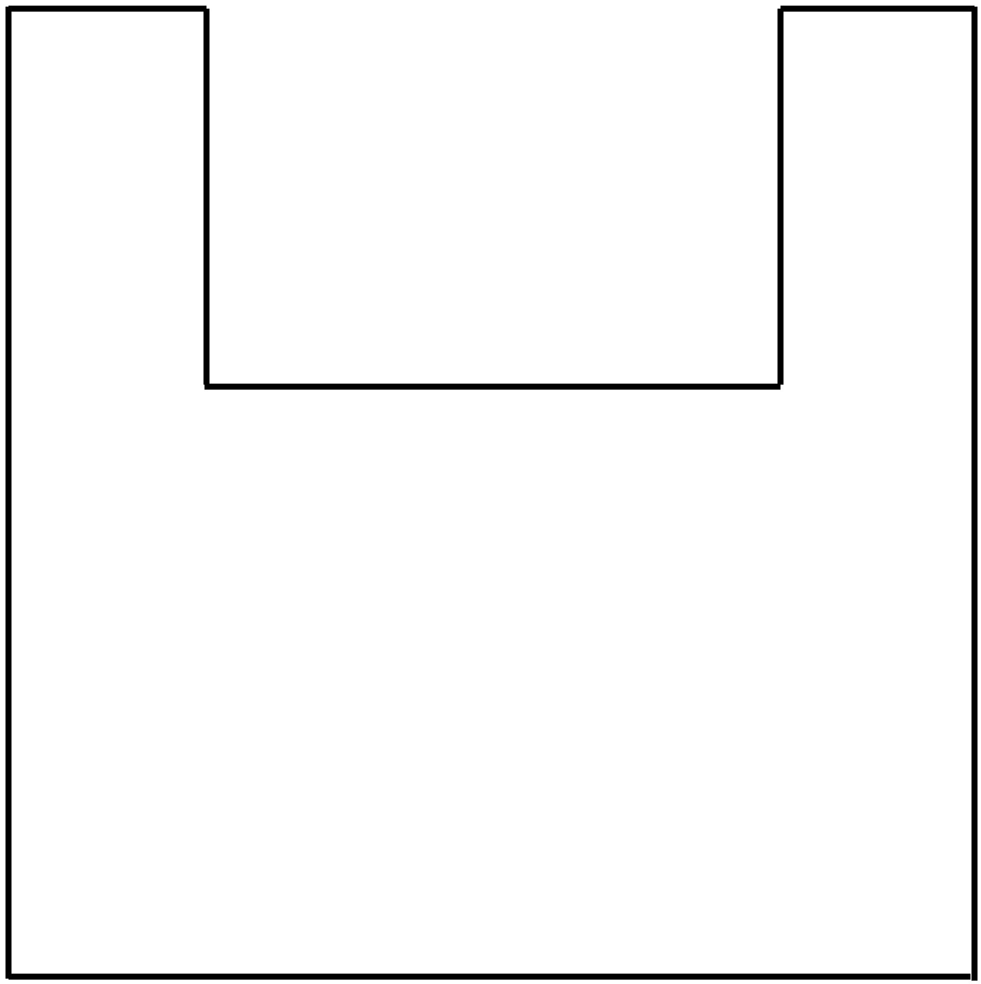
3) Think about area.
   (a) When might you need to use area in your everyday life?
   (b) Give an example of when estimating an area is useful.
   (c) Give an example of when measuring an area is necessary.

4) Think about perimeter.
   (a) When might you need to use perimeter in your everyday life?
   (b) Give an example of when estimating a perimeter is useful.
   (c) Give an example of when measuring a perimeter is necessary.

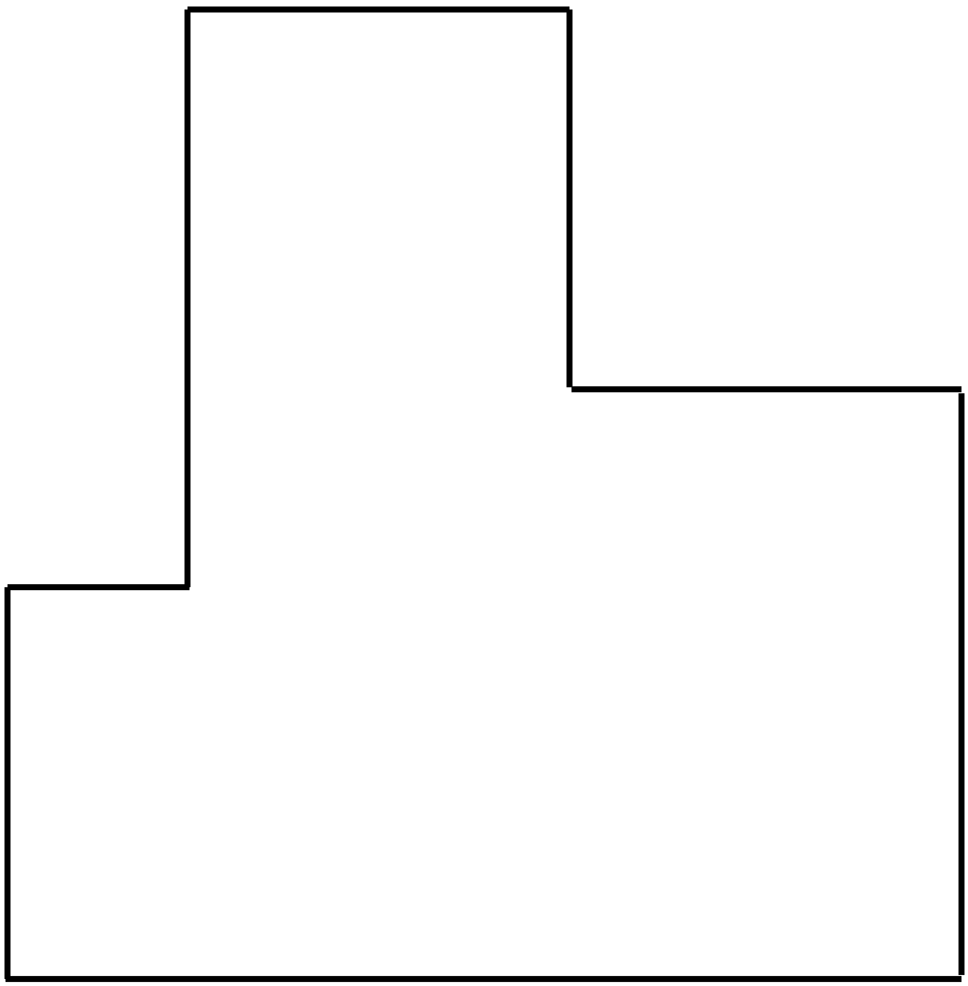
Shape I



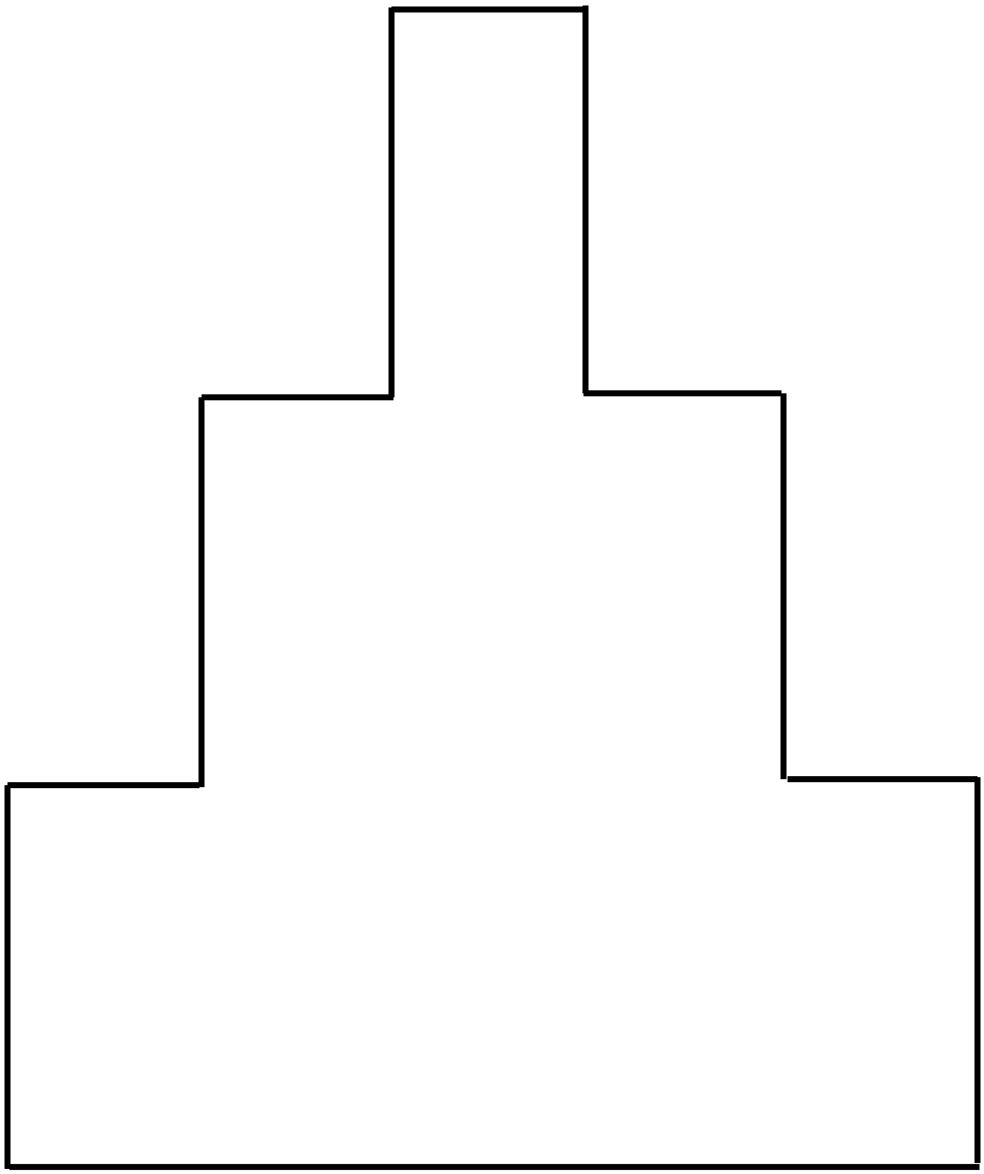
Shape II



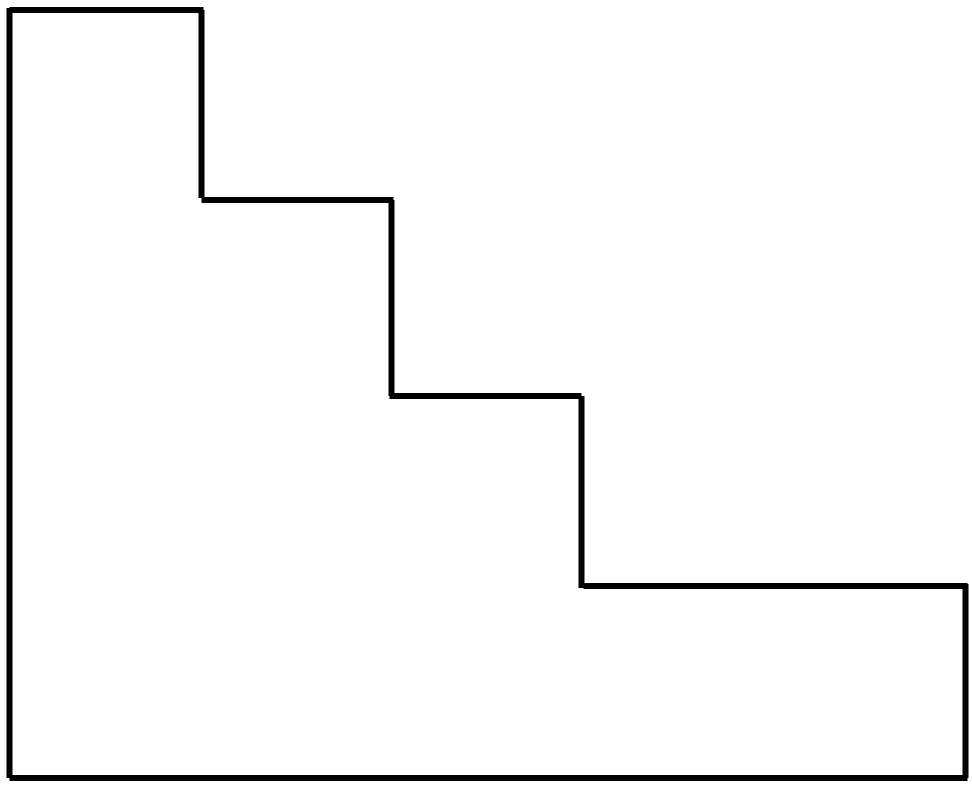
Shape III



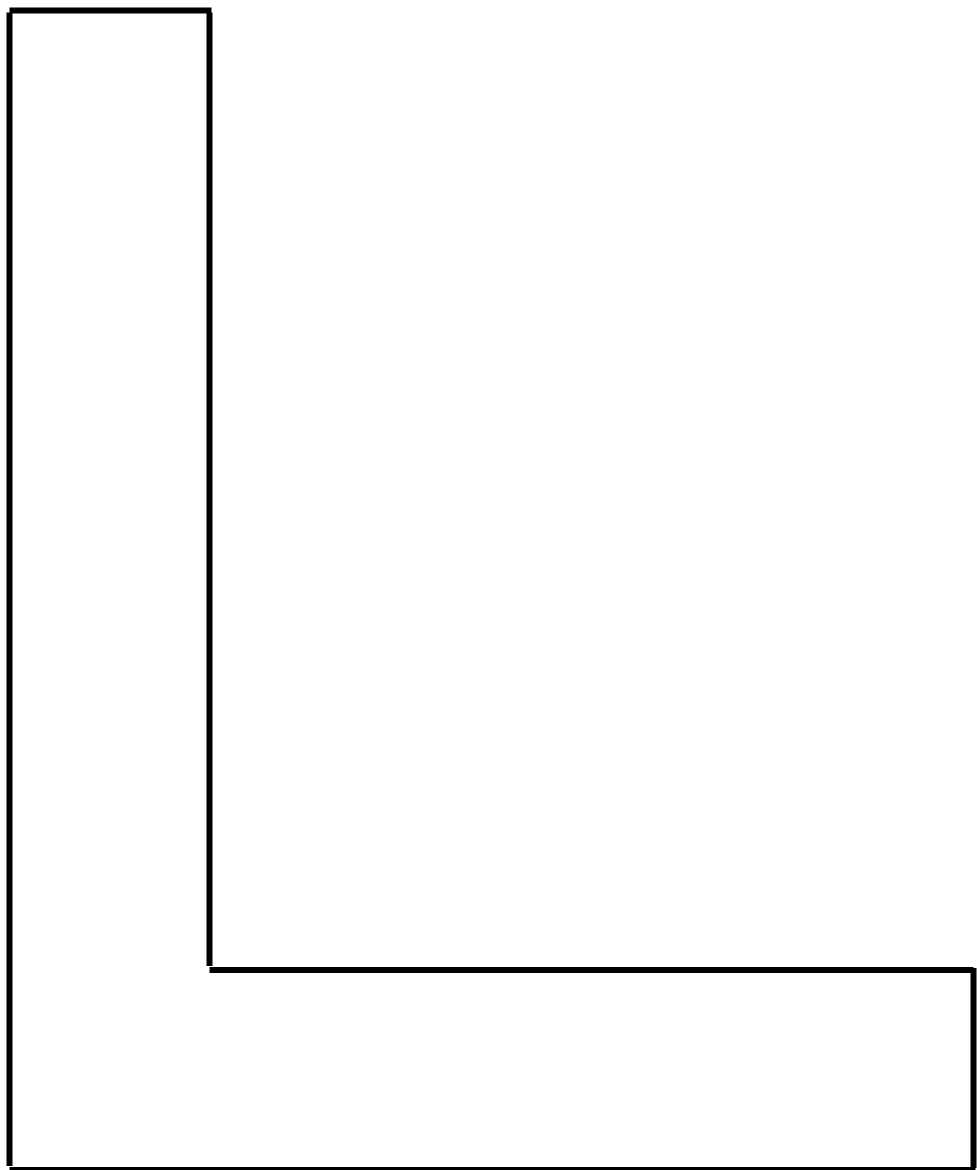
Shape IV



Shape V

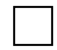


Shape VI

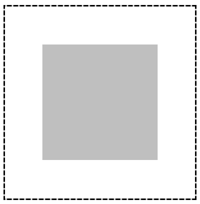


Manipulative Mathematics **Name_____________________**

Measuring Area and Perimeter – Extra Practice

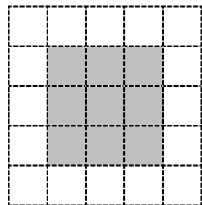
Find the area and perimeter of each shaded region, using this square  as one square unit measure.

1)



Estimated area ______

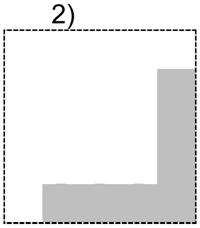
Estimated perimeter ____



Measured area ______

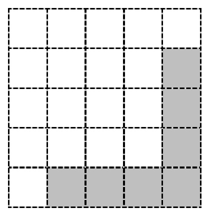
Measured perimeter ____

2)



Estimated area _____
M

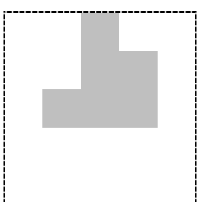
Estimated perimeter ____



Measured area _____

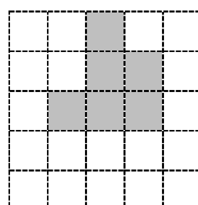
Measured perimeter ___

3)



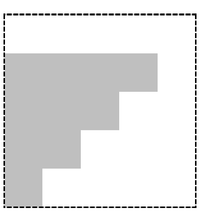
Estimated area _____

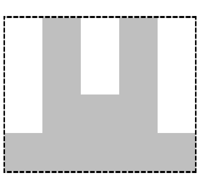
Estimated perimeter ___



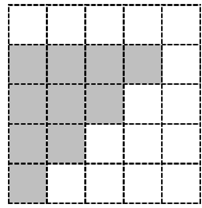
Measured area _____

Measured perimeter ___





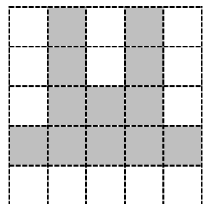
4)



Estimated area _____ Measured area _____

Estimated perimeter ___ Measured perimeter ___

5)



Estimated area _____ Measured area _____

Estimated perimeter ___ Measured perimeter ___

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Coin Lab

Manipulatives used:

Plastic coins

Teacher video: Teaching Mixture Applications with A Coin Lab Activity

Coin Lab

Resources Needed:

Each group of students needs a bag containing a handful of several different kinds of plastic coins (pennies, nickels, dimes, etc.).

Background Information:

Students often have trouble with coin word problems. They get confused with all the information given in the problem and don't know how to even begin solving it. Part of the difficulty stems from the fact that they don't distinguish between the *number* of coins and the *value* of the coins. By using a concrete example familiar to them from everyday life, students develop a strategy for calculating the value of a handful of coins. The strategy directly leads them to an organized method for solving coin word problems. This exercise may seem almost trivial to teachers, but it can make a tremendous difference in students' ability to solve coin word problems.

Directions:

- This activity is best done with students in groups of 2 to 4.
- Give each group the worksheet and a bag of coins. It is not necessary for the all the bags to contain the same number of coins.
- On the worksheet, students are asked to determine the value of all the 'money' in their bag. Students need to devise a method for counting the money.
- After finding the total value of their 'money', they must describe completely, in words, the method they used. They must write their method step-by-step.

  Many students find this hard to do–you might want to tell them to start with "dump the coins from the bag to the desk" and then list each step carefully. Walk around and notice if they are stuck on vocabulary. Often they will say "sort coins in piles." Help them realize that they sorted the piles by *type of coin* or by the *value of the coin*. Often they next say "add them all up." They need to distinguish counting the number of coins and the total value of coins.

  Most students eventually devise a method similar to this:

  1) Dump the coins from the bag.
  2) Sort the coins into piles based on the type of coin—quarters, dimes, nickels, etc.
  3) Count the number of coins in each pile.
  4) Find the value of each pile by multiplying the number of coins times the value of that type of coin.
  5) Add together the value of all the piles to get the total value of all the coins.

  It is steps 3, 4, and 5 that will lead them to an organized strategy for solving coin word problems.

- Finally, students must show the calculations for their bag of coins. For example:
  - 3 quarters - $3 \times 0.25 = 0.75$
  - 9 dimes - $9 \times 0.10 = 0.90$
  - 13 nickels - $13 \times 0.05 = 0.65$

- 27 pennies - $27 \times 0.01 = \underline{0.27}$

  $2.57

- While the groups are working, you might want to ‘gather data’ to create a few coin word problems using your students’ names and some of the coins on their desks. Just choose 2 or 3 types of coins. For example, if Melissa has 8 nickels and 6 dimes (ignoring the rest of her coins), you could write on the board:

  > Melissa has nickels and dimes worth a total of $1.00. The number of nickels is two more than the number of dimes. How many nickels and how many dimes does Melissa have?

  If you do this, try to cover all the different types of coins with your problems. You might build up the complexity of the problems by starting with two types of coins that you relate to each other using ‘more than’, then ‘less than’, then ‘twice as many’, and finally using three or more types of coins.

- You may wish to have a few groups write on the board the methods and the calculations they used to find the total value of their coins. If a group used another method instead of sorting by type of coin, (for example, some students sort coins into piles that add up to $1) have them explain their method, and then ask the class to find the total value by the ‘number x value’ method.

- This can be a good time to introduce a chart, such as the one shown below, to help students organize all the information. The chart will be especially helpful when doing coin word problems, and can easily be adapted for other mixture problems.

Type of coin	**Number**	**Value**	**Total value**

Students can get more practice finding the value of a handful of coins by choosing ‘Count’ in the NCTM Illuminations Coin Box activity. In this activity, several coins appear scattered around the workspace. Students can group like coins by dragging them together. There is even an option to display the value of each coin; this might be especially helpful for students who are not completely familiar with U.S. coins. Finally, students enter the total value of the coins and click to check their answer. This website for this interactive activity is http://illuminations.nctm.org/ActivityDetail.aspx?id=217.

Manipulative Mathematics **Name**______________________________

Coin Lab

Team Members:____

1) How much 'money' is in your bag?

2) Describe, in words, **the method** you used to determine how much 'money' is in your bag. List everything you did, step-by-step, so that someone not in your group could follow your directions.

3) Show the **calculations** you used to determine the total value of the money in your bag.

Manipulative Mathematics **Name**______________________

Coin Lab - Practice

Fill in the charts to calculate the total value of each set of coins.

1)

Type of coin	Number	Value ($)	Total value ($)
Quarters	3	0.25	
Nickels	4	0.05	
			$0.95

The total value of all the coins goes here.

2)

Type of coin	Number	Value ($)	Total value ($)
Dimes	5	0.10	
Pennies	1	0.01	

3)

Type of coin	Number	Value ($)	Total value ($)
Dimes	8		
Nickels	11		

4)

Type of coin	Number	Value ($)	Total value ($)
Quarters	6		
Pennies	9		

5)

Type of coin	Number	Value ($)	Total value ($)
Pennies	7		
Quarters	9		
Dimes	4		

6)

Type of coin	Number	Value ($)	Total value ($)
Nickels	15		
Pennies	3		
Dimes	8		

7)

Type of coin	Number	Value ($)	Total value ($)
Pennies	19		
Quarters	13		
Nickels	22		

8)

Type of coin	Number	Value ($)	Total value ($)
Dimes	12		
Nickels	19		
Quarters	16		

9)

Type of coin	Number	Value ($)	Total value ($)
Pennies	24		
Nickels	17		
Dimes	31		
Quarters	15		

10)

Type of coin	Number	Value ($)	Total value ($)
Pennies	29		
Nickels	14		
Dimes	23		
Quarters	35		

For more practice finding the value of a 'handful' of coins go to the website http://illuminations.nctm.org/ActivityDetail.aspx?id=217 and choose 'Count'. If you are not completely familiar with U.S. coins, you may find it helpful to have the program display the value of each coin next to its picture.

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Slopes

Exploring Slopes of Lines

Slope of Line Between Two Points

Manipulatives used:

Geoboards

Teacher video: Teaching Slope Concepts

Exploring Slopes of Lines

Resources Needed:

Each student needs a worksheet, a geoboard, and 3 or more rubber bands. If geoboards are not available, you can photocopy the last page of this packet.

Background Information:

Slope is a concept for which doing a simple concrete exercise may make a substantial difference in student understanding. This activity introduces students to the $\frac{\text{rise}}{\text{run}}$ definition of slope. Students will model a line on a geoboard with a rubber band, then form a right triangle and count the rise and the run to determine the slope of the line. For students new to the concept of slope, it helps to consistently work from the left of the geoboard to the right and find the rise first and then the run, in the same order they will put the numbers into the formula $m = \frac{\text{rise}}{\text{run}}$. Also, when working from left to right, the run is always positive, while the rise is positive or negative, depending on whether the rubber band is stretched up or down. 'Reading' a graph from left to right will also be consistent with the concepts of increasing or decreasing functions, which students encounter in higher level math classes. By completing this activity as well as "Slopes of Lines Between Two Points" students gain comprehension of this abstract topic.

Directions:

- Pass out the geoboards and rubber bands. Ideally, each student should have a geoboard, but the activity can be done with students working in pairs.
- Work through Exercise 1 with the whole class. Begin by demonstrating how to represent a line on the geoboard with a rubber band, then stretch the rubber band up and to the right to form a right triangle as shown in 1(b). Emphasize that the sides of the right triangle are vertical and horizontal lines. Discuss the definition of slope, then count the rise and the run in your triangle—make sure students watch you count the spaces, not the pegs. In the figure for 1(b) the rise is 6 and the run is 5.
- Next, model a line with negative slope on your geoboard. Show students how the rise is negative, but the run, still counting left to right, is positive. (Notice that in Exercise 5(b) we put the negative sign in the numerator.)
- Then let the students proceed through the worksheet. Whether they are working individually or with a partner, encourage them to compare their geoboard lines with those of their neighbors to promote dialog.
- As students work, you may want to spot check their answers to Exercises 2, 3, and 4, which are open-ended. A common error students make is to count the pegs, instead of the spaces.
- Students can get additional practice using virtual geoboards to model lines and triangles and explore slopes at the National Library of Virtual Manipulatives website http://nlvm.usu.edu/en/nav/frames_asid_279_g_4_t_3.html?open=activities&hidepanel=true&from=topic_t_3.html.

Manipulative Mathematics **Name**______________________

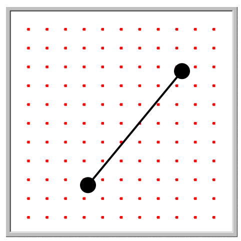
Exploring Slopes of Lines

The concept of slope has many applications in the real world. The pitch of a roof, grade of a highway, ramp for a wheelchair are some places you literally see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

We will use geoboards to explore the concept of slope. Using rubber bands to represent lines and the pegs of the geoboards to represent points, we have a concrete way to model lines on a coordinate grid. By stretching a rubber band between two pegs on a geoboard, you'll discover how to find the slope of a line.

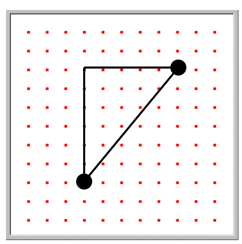
1) Let's work together to see how to use a geoboard to find the slope of a line.

(a) Take your geoboard and a rubber band. Stretch the rubber band between two pegs like this:



Doesn't it look like a line?

(b) Now stretch the rubber band straight up from the left peg and around a third peg to make the sides of a right triangle, like this:



Be sure to make a 90º angle around the third peg, so one of the two newly formed lines is vertical and the other side is horizontal. You have made a right triangle!

To find the slope of the line count the distance along the vertical and horizontal sides of the triangle. The vertical distance is called the **rise** and the horizontal distance is called the **run**.

> **Slope**
>
> The **slope** of a line is $m = \dfrac{\text{rise}}{\text{run}}$
>
> **rise** measures the vertical change $\updownarrow$
>
> **run** measures the horizontal change $\leftrightarrow$

(c) On your geoboard, what is the rise? _____________

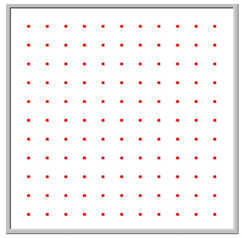
(d) What is the run? _____________

(e) What is the **slope** of the line on your geoboard?

$$m = \frac{\text{rise}}{\text{run}}$$

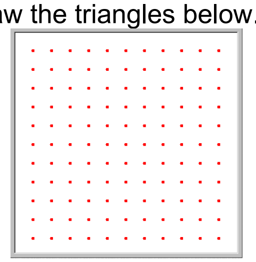
$$m = \frac{\square}{\square}$$

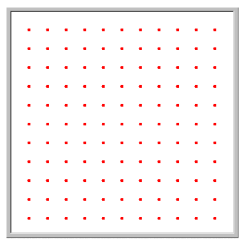
2) Make another line on your geoboard, and form its right triangle. Draw a picture of your geoboard here:

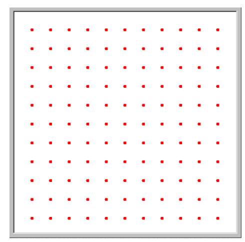


(a) What is the rise? _____ (b) What is the run? ______ (c) What is the slope? ______

3) Make 3 more lines on your geoboard, form the right triangle for each, and count their slopes. Draw the triangles below.





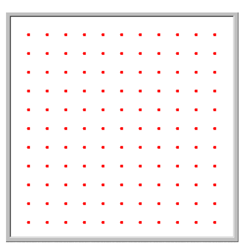


(a) Slope = _________ (b) Slope = ________ (c) Slope = _________

4) If the left endpoint of a line is higher than the right endpoint, you have to stretch the rubber band down to make the right triangle. When this happens the rise will be negative because you count down from your starting peg.

(a) Do any of your lines in exercise 3 have negative slope? ___________

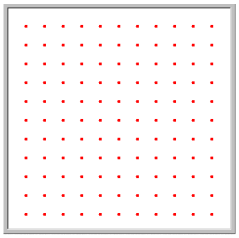
(b) Draw a line with negative slope here and calculate its slope:



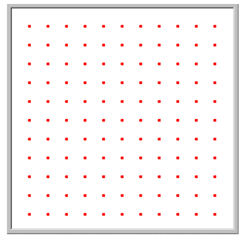
Slope = _________

5) Use a rubber band on your geoboard to make a line with each given slope and draw a picture of it.

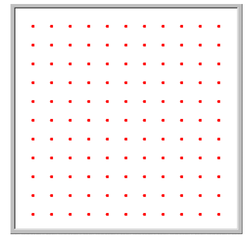
(a) Slope = $\frac{1}{3}$



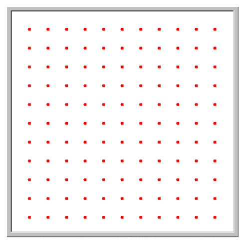
(b) Slope = $\frac{-3}{4}$



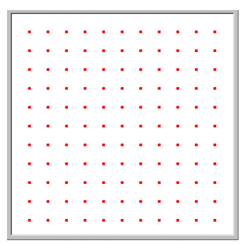
(c) Slope = 2 (hint: $2 = \frac{?}{?}$)



6) Make a horizontal line on your geoboard and draw it here. What is the slope of the horizontal line?



7) Make a vertical line on your geoboard and draw it here. What is the slope of the vertical line?



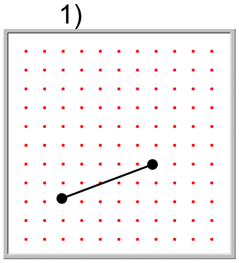
Manipulative Mathematics **Name______________________**

Exploring Slopes of Lines- Extra Practice

Sketch the rise and the run for the line modeled on each geoboard, then calculate the slope of the line.

You may want to use the virtual geoboard online at http://nlvm.usu.edu/en/nav/frames_asid_279_g_4_t_3.html?open=activities&hidepanel=true&from=topic_t_3.html.

1)

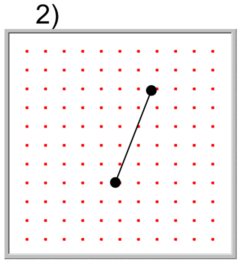


(a) rise = ____

(b) run = ____

(c) slope = ____

2)

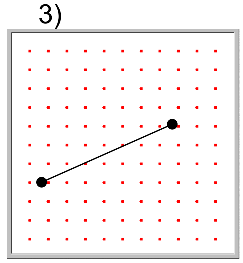


(a) rise = ____

(b) run = ____

(c) slope = ____

3)

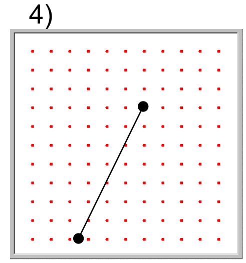


(a) rise = ____

(b) run = ____

(c) slope = ____

4)

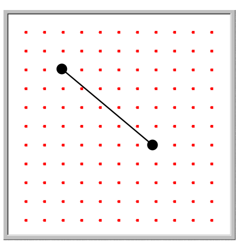


(a) rise = ____

(b) run = ____

(c) slope = ____

5)

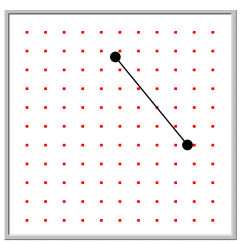


(a) rise = ____

(b) run = ____

(c) slope = ____

6)

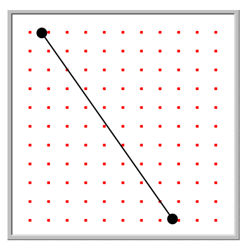


(a) rise = ____

(b) run = ____

(c) slope = ____

7)

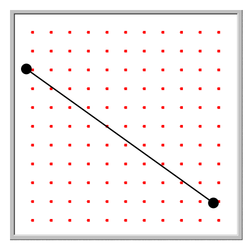


(a) rise = ____

(b) run = ____

(c) slope = ____

8)



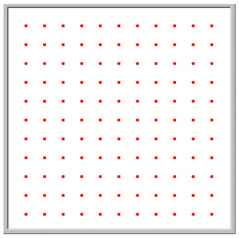
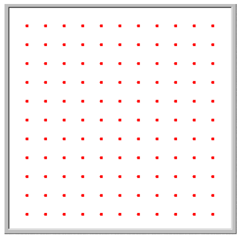
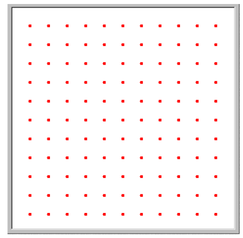
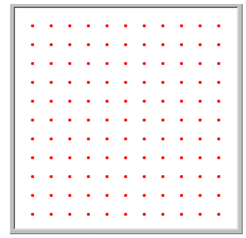
(a) rise = ____

(b) run = ____

(c) slope = ____

Draw a line with the given slope.

9) slope = $\frac{3}{10}$ 10) slope = $\frac{8}{5}$ 11) slope = $\frac{-1}{6}$ 12) slope = $\frac{-7}{4}$

Slope of Lines Between Two Points

Resources Needed:

Each student needs a worksheet, a geoboard, and 3 or more rubber bands. If geoboards are not available, you can photocopy the last page of this packet.

Background Information:

Slope is a concept for which doing a simple concrete exercise may make a substantial difference in student understanding. In this activity, students will model a small coordinate system on a geoboard. They will locate pairs of points on the coordinate system, connect them with a rubber band to model a line, and then count the rise and run to calculate the slope.

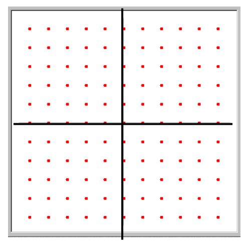
Directions:

- Working in pairs is best to promote student dialogue, but, if possible, each student should have his or her own geoboard.
- This activity should be used following "Exploring Slopes of Lines". (The two activities may be done during the same class or on separate days.)
- After discussing "Exploring Slopes of Lines" with the class and assessing their understanding, demonstrate how to create a coordinate grid on the geoboard by stretching two rubber bands to form the x - axis and the y - axis. Spend a few minutes having students locate basic points.
- Demonstrate one example of finding the slope of a line between two points. You may wish to start by modeling a line segment with a rubber band, then identifying the coordinates of its endpoints, and then counting the rise and the run.
- Have the class do the worksheet. You may need to prompt students to "count backwards" in order to do Exercise 9.
- Notice that this activity uses the definition of slope $m = \frac{rise}{run}$. If you think your students are ready for it, you may wish to introduce the $\frac{y_2 - y_1}{x_2 - x_1}$ formula after the class has completed the worksheet.
- The National Library of Virtual Manipulatives has an online geoboard with axes at http://nlvm.usu.edu/en/nav/frames_asid_303_g_4_t_3.html?open=activities&hidepanel=true&from=topic_t_3.html.

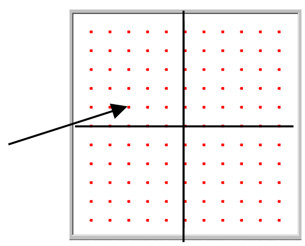
Manipulative Mathematics **Name________________________**

Slope of Lines Between Two Points

1) Start with a geoboard and 2 rubber bands. Stretch one rubber band around the middle row of pegs horizontally and the other rubber band around the middle row of pegs vertically to model the x - axis and the y - axis, like this:

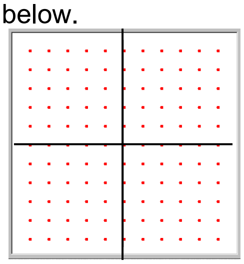


You now have a small coordinate system, with $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$. Each of the pegs on the geoboard represents a point on the graph. For example, the point $(-3,1)$ is located at the arrow.



2) On your geoboard, make a line between the points $(-3,1)$ and $(4,3)$.

(a) Sketch it on the geoboard below.



(b) To find the rise and the run, stretch the rubber band into a right triangle, with one side vertical and the other horizontal. Draw your triangle on the geoboard above.

(c) What is the rise? ____________________

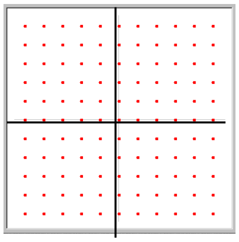
(d) What is the run? ____________________

(e) The slope is $m = \dfrac{rise}{run}$.

$$m = \frac{\square}{\square}$$

Find the slope of the line between each pair of points. Use your geoboard with a rubber band to model each line, then form a right triangle to find the rise and the run. Sketch each model.

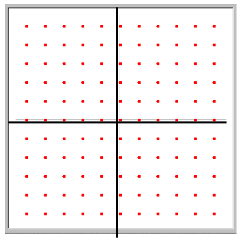
3) $(-3, 0)$ & $(1, 5)$



Slope= $\frac{\text{rise}}{\text{run}}$

$m = \frac{\square}{\square}$

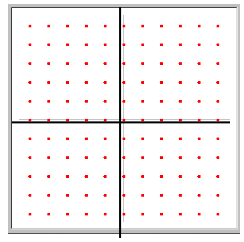
4) $(-2, -4)$ & $(0, 3)$



Slope= $\frac{\text{rise}}{\text{run}}$

$m = \frac{\square}{\square}$

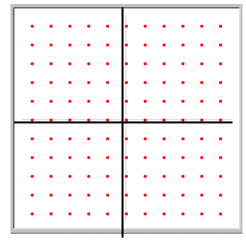
5) $(-1, 2)$ & $(4, -1)$



Slope= $\frac{\text{rise}}{\text{run}}$

$m = \frac{\square}{\square}$

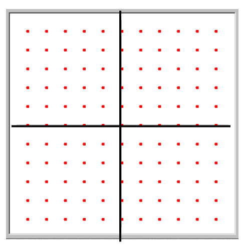
6) $(-3, -2)$ & $(-2, -5)$



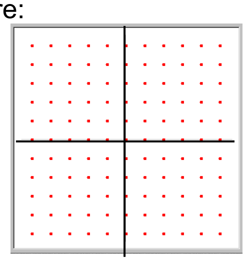
Slope= $\frac{\text{rise}}{\text{run}}$

$m = \frac{\square}{\square}$

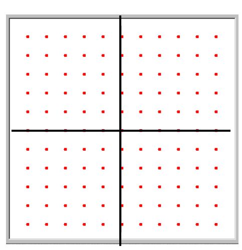
7) Start at the point $(-1, -1)$ and make a line with slope $\frac{3}{2}$ by counting the rise (up 3) and the run (over 2). Draw the line here:



8) Start at the point $(2, 1)$ and make a line with slope $\frac{-1}{3}$ by counting the rise (down 1) and the run (over 3). Draw the line here:



9) Start at the point $(4, 4)$ and make a line with slope $\frac{3}{4}$ by counting the rise and the run. Draw the line here:



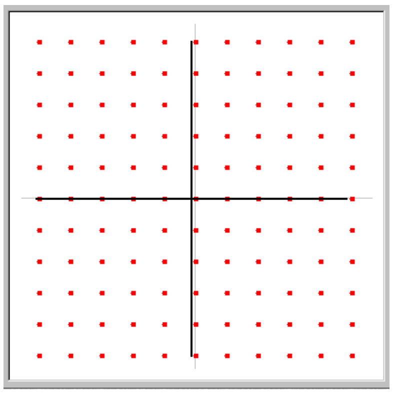
Manipulative Mathematics **Name_______________________**

Slope of Lines Between Two Points – Extra Practice

Draw the line between each pair of points and then find its slope. You may wish to sketch a right triangle for each line to help you count the rise and the run.

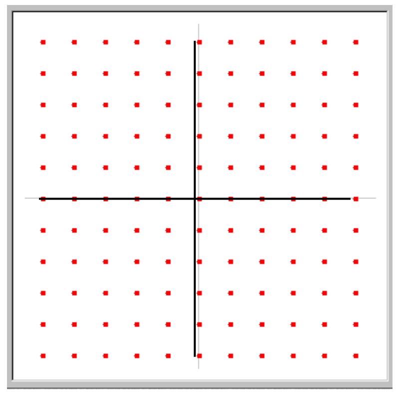
You may want to use the interactive geoboards at the website:
http://nlvm.usu.edu/en/nav/frames_asid_303_g_4_t_3.html?open=activities&hidepanel=true&from=topic_t_3.html.

1) $(-4, 0)$ and $(0, 5)$



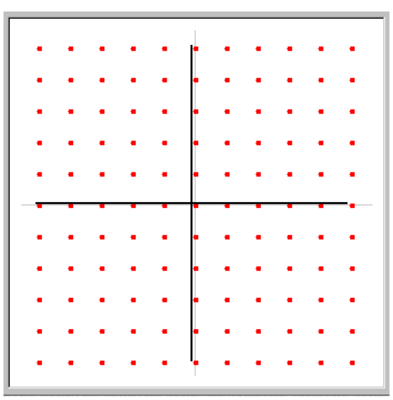
slope = ____________

2) $(0, -3)$ and $(2, 0)$



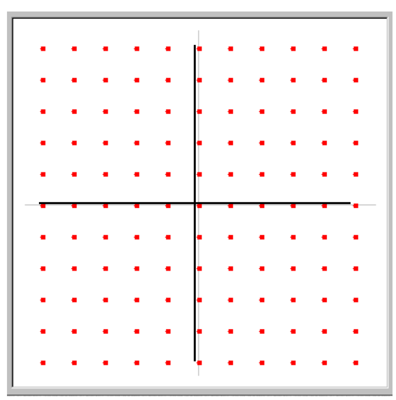
slope = ____________

3) $(-2, -3)$ and $(1, 1)$



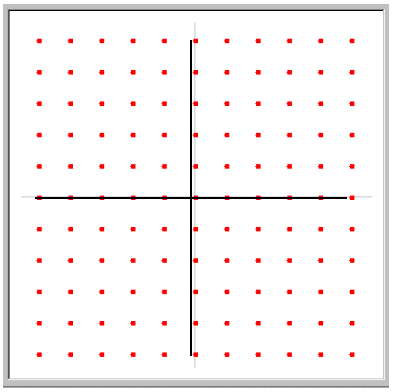
slope = ____________

4) $(-5, 2)$ and $(4, 3)$



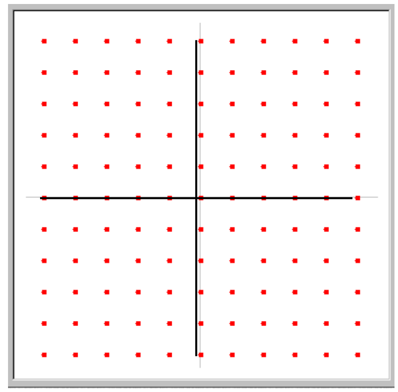
slope = ____________

5) $(-1, 4)$ and $(5, -3)$



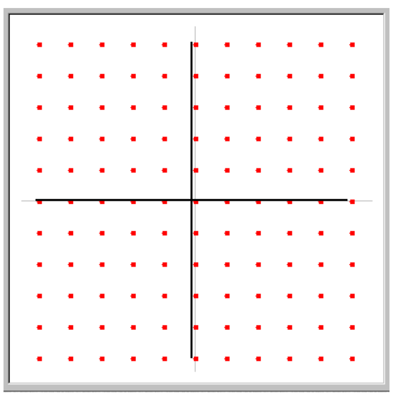
slope = ____________

6) $(-4, -2)$ and $(4, -5)$



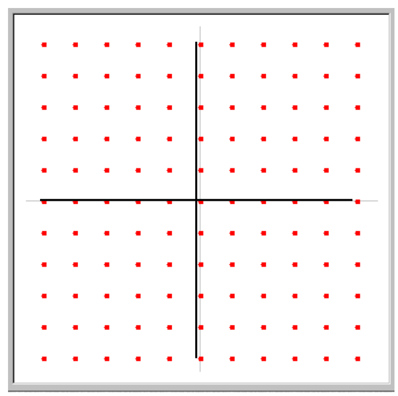
slope = ____________

7) $(-3, 2)$ and $(1, 2)$



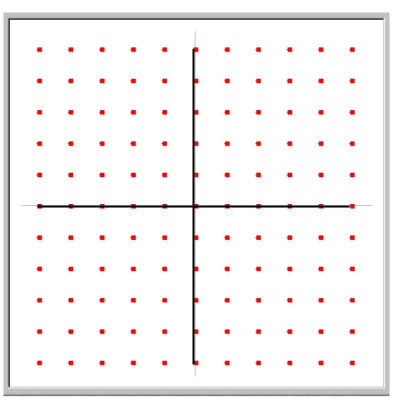
slope = ____________

8) $(5, -5)$ and $(5, 3)$

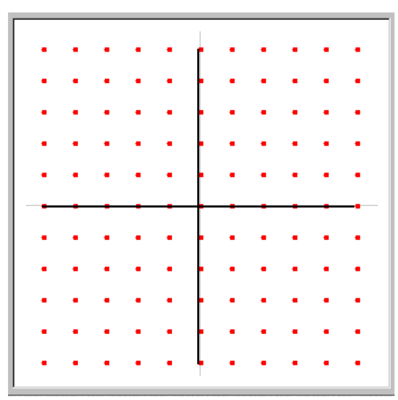


slope = ____________

9) Starting at $(-4, -3)$ sketch a line with slope $\frac{5}{3}$.



10) Starting at $(-2, 5)$ sketch a line with slope $\frac{-9}{7}$.



Manipulative Mathematics

Geoboard Template