MANIPULATIVE MATHEMATICS FOR STUDENTS

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Pre-alg-ebra

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Prealgebra Concepts

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To the Student:

Research has shown that our brains learn best when we start with concrete objects and then move on to abstract ideas. Manipulatives are concrete objects used to model abstract mathematical concepts. Although the word 'manipulatives' may be new to you, using manipulatives is probably something you have already done. If you learned to add numbers by counting your fingers, you were using manipulatives. And maybe you used two-color counters or fraction tiles in a previous math class.

You will see Illustrations of manipulatives in ***Prealgebra*** whenever a new concept is introduced. Then you will be encouraged to do a ***Manipulative Mathematics*** activity to help you develop a solid understanding of the concept.

This booklet contains the ***Manipulative Mathematics* activities** that accompany ***Prealgebra***. Each activity relates to a mathematical topic covered in ***Prealgebra***. There are two parts to each activity:

- The **student worksheet** will lead you through a series of questions to help you understand a mathematical property or procedure through the use of manipulatives.
- The **extra practice problems** reinforce the results of the student worksheet. Most Extra Practice worksheets include a link to a website with online (or 'virtual') manipulatives.

Ask your math instructor if your college has manipulatives for you to use. You can access the virtual manipulatives online any time, 24/7. We have sets of manipulatives, such as color chips, fraction pieces, algebra tiles, geoboards, and more, in our classrooms. Our college students use them to model and learn about critical mathematical concepts and procedures. We have heard many "a-ha's" from students who finally understand, for example, how to work with fractions or signed numbers.

We sincerely hope you will use our ***Manipulative Mathematics* activities** to help you succeed in ***Prealgebra***. We wholeheartedly believe that through the use of manipulatives, you can develop an understanding of mathematics that translates into success throughout your mathematics courses.

Lynn Marecek MaryAnne Anthony-Smith

Manipulative Mathematics **Name**______________________

Game of 24

The Game of Twenty-four is a great way to think mathematically.
Given four numbers, you add, subtract, multiply and/or divide them so that the result is 24. You must use each number once--but <u>only</u> once.

Start with the numbers 1, 1, 4, and 8.

1) How can you use these numbers to create 24? Don't worry yet about 1, 1, 4, and 8. Think of pairs of any two numbers that multiply to 24. List some of the pairs here:

2) First, let's think of 24 as the product of $3 \bullet 8$.
   We want to combine **1, 1, 4, 8** to get 3 and 8.

   (a) One way is to use 4 minus 1 to get 3, then 3 times 8 is 24. But we need to use the number 1. How can we use the 1 and still have 24? 24 times 1 is still 24.

   Putting this all these steps together using good algebra notation gives $4-1\ (8)(1)$.
   Verify that this expression simplifies to 24.

   $$4-1\ \ 8\ \ 1$$

   (b) Here is another way to use the same four numbers, 1, 1, 4, 8, to get the product $3 \bullet 8$:
   4 times 1 is 4, and then 4 minus 1 gives 3. Finally multiply that 3 by 8 to get 24.
   Show that this expression simplifies to 24:

   $$4 \bullet 1-1\ \ 8$$

3) This time, we'll use the fact that 24 is the product of $6 \bullet 4$.

   (a) Can we combine 1, 1, 4, 8 to make 6 times 4?
   Well, 1 plus 1 is 2, 8 minus 2 gives 6, and then 6 times 4 is 24.
   Show that this expression simplifies to 24:

   $$\left[8-\ 1+1\right] \bullet 4$$

   (b) Can you think of another combination? Using good algebra notation, write a different expression and show that it simplifies to 24.

4) Another number fact that might help make 24 is $12 \bullet 2 = 24$.

(a) How can you combine 1, 1, 4, 8 to create 12 and 2?
4 plus 8 is 12, and 1 plus 1 is 2. Then twelve times two is 24!
Write this as one expression using good algebra notation, then show that it simplifies to 24.

(b) Can you think of another combination? Using good algebra notation, write a different expression and show that it simplifies to 24.

Now use the numbers 5, 3, 5, 4 to make 24.

5) Verify that each expression simplifies to 24.

(a) $5 \bullet 5 + 3 - 4$

(b) $3 \bullet 5 + 5 + 4$

6) Using good algebra notation, write a different expression that simplifies to 24.

Next try 3, 6, 6, 9.

7) Verify that each expression simplifies to 24.

(a) $3 + 6 + 6 + 9$

(b) $6 \bullet 9 \div 3 + 6$

8) Using good algebra notation, write a different expression that simplifies to 24.

Manipulative Mathematics **Name**______________________

Game of 24 – Extra Practice

For each set of numbers use good algebra notation to write 2 different expressions that simplify to 24.

1) 1, 2, 3, 4

(a) (b)

2) 1, 2, 5, 9

(a) (b)

3) 1, 1, 7, 8

(a) (b)

4) 1, 7, 8, 9

(a) (b)

5) 2, 4, 6, 6

(a) (b)

6) 2, 3, 3, 6

(a) (b)

7) 2, 2, 4, 5

(a) (b)

8) 3, 3, 4, 5

(a) (b)

9) 3, 4, 5, 7

(a) (b)

10) 3, 4, 7, 9

(a) (b)

For more practice, there are several websites where you can play the Game of Twenty-four online. One of them is http://www.mathplayground.com/make_24.html
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Manipulative Mathematics **Name**______________________

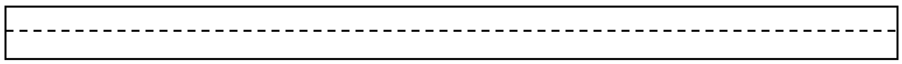
Number Line

The Number Line Part 1 -- Counting Numbers and Whole Numbers

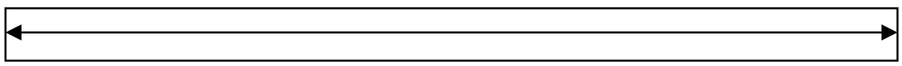
Counting numbers and whole numbers can be visualized by creating a **number line**.

1) To create your own number line:

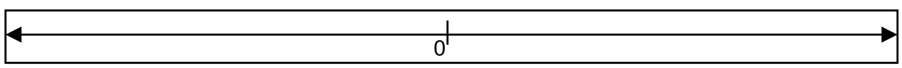
(a) Take a strip of paper about 3 feet long and fold it lengthwise to make a straight crease.



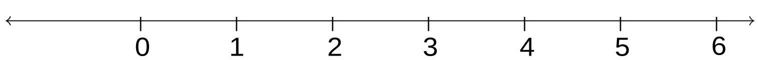
(b) Open the fold and draw a line in the crease. Put an arrow at each end of the line to indicate that the line continues.



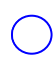
(c) Mark a point at about the middle of the line. Label that point 0. This point is called the **origin**.



2) Choose a convenient unit and mark off several of these units to the right of 0. Pair these points with the numbers 1, 2, 3, 4, 5, … and so on. When a number is paired with a point, we call it the **coordinate** of the point.



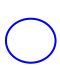
3) Draw a red triangle around each counting number. 

4) Draw a blue circle around each whole number. 

5) Notice that all the numbers on your number line except 0 are marked with both a triangle and a square. What conclusion can you draw from this?

6) In one corner of your strip make a “key” that explains the symbols around the numbers.

 Counting numbers

 Whole numbers

7) Put your number line in your notebook for future use, so you can add more numbers to the number line as you proceed through this course.

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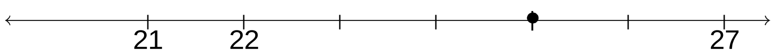
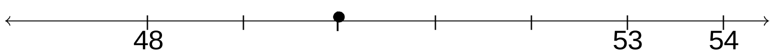
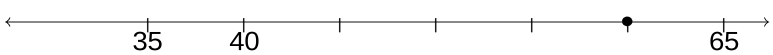
Number Line Part I – Extra Practice

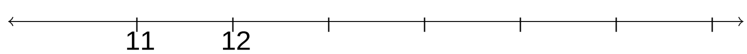
Name the coordinate of each point.

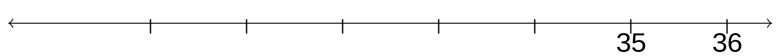
1) 

2) 

3) 

Locate each point on the number line.

4) 15 

5) 31 

6) 125 

For each set of numbers identify (a) the counting numbers and (b) the whole numbers.

7) $0, \frac{1}{5}, 4, 7.5, 23, 199$ (a) ________________ (b) ________________

8) $0, \frac{3}{4}, 1, 5\frac{1}{2}, 16, 99.9, 250$ (a) ________________ (b) ________________

9) $0, \frac{2}{9}, 3.1, 6, 10\frac{1}{4}, 88, 132.5$ (a) ________________ (b) ________________

10) $0, 1, \frac{5}{2}, 5.2, 8, 24.99, 165, 200$ (a) ________________ (b) ________________

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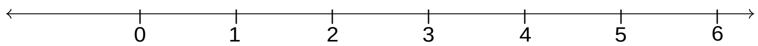
Number Line

The Number Line Part 2 -- Integers

The number line you made in Part 1 started at 0. All the numbers you have worked with so far have been positive numbers, numbers greater than 0.

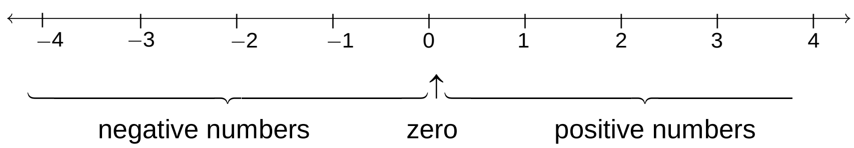


Now you need to expand your number line to include negative numbers, too. Negative numbers are numbers less than zero. So the negative numbers will be to the left of zero on the number line.

Get your number line out of your notebook and place it on your desk.

1) Mark off several units to the **left** of zero. Make sure your unit is the same size as the one you used on the positive side.

2) Now label -1 at the first point left of 0, then -2 at the next point to the left, and so on.



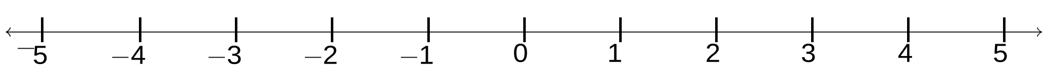
3) The arrows on both ends of the number line indicate that the numbers keep going forever.

   (a) Is there a largest positive number? ____ (b) Is there a smallest negative number? ____

4) Is zero a positive or a negative number?______________ Numbers larger than zero are positive and numbers smaller than zero are negative. Zero is neither positive nor negative.

5) Locate and label the following points on this number line.

   (a) 2 (b) -1 (c) -4 (d) 5 (e) -5



The whole numbers and their opposites are called the **integers**.

Integers: ... -3, -2, -1, 0, 1, 2, 3,...

6) Put a black square around each integer on your number line. ☐

7) What do you notice about the integers, counting numbers and whole numbers on your number line?

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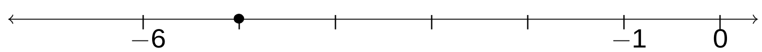
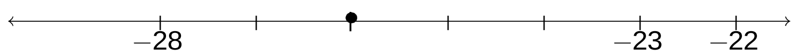
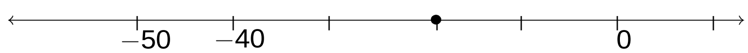
Number Line Part 2 – Extra Practice

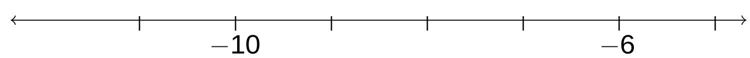
Name the coordinate of each point.

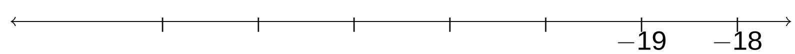
1) 

2) 

3) 

Locate each point on the number line.

4) -9 

5) -22 

6) -65 

For each set of numbers identify the (a) counting numbers, (b) whole numbers, and (c) integers.

7) $-3,\ -\frac{1}{2},\ 0,\ \frac{9}{10},\ 5,\ 7.5,\ 32$ (a) ________ (b) ________ (c) ________

8) $-12,\ -\frac{3}{4},\ 0,\ 2,\ 4.65,\ 29,\ 48\frac{1}{6}$ (a) ________ (b) ________ (c) ________

9) $-8.2,\ -3,\ -\frac{5}{9},\ 0,\ 4,\ \frac{26}{3},\ 99$ (a) ________ (b) ________ (c) ________

10) $-\frac{15}{4},\ -2.5,\ -1,\ 0,\ \frac{4}{7},\ 10,\ 28.1$ (a) ________ (b) ________ (c) ________

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Number Line

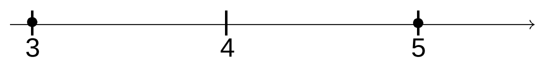
The Number Line Part 3 -- Fractions

Now you are ready to include fractions on your number line. This will help you visualize fractions and understand their value. Take your number line out of your notebook and place it on your desk.

Our goal is to locate the numbers $\frac{1}{5}$, $\frac{4}{5}$, 3, $3\frac{1}{3}$, $\frac{7}{4}$, $\frac{9}{2}$, 5, and $\frac{8}{3}$ on the number line.

1) We'll start with the whole numbers 3 and 5 because they are the easiest to plot.

Put points to mark 3 and 5.



2) The proper fractions listed are $\frac{1}{5}$ and $\frac{4}{5}$.

(a) Proper fractions have value less than one. Between which two whole numbers are the proper fractions $\frac{1}{5}$ and $\frac{4}{5}$ located? They are between _____ and _____.

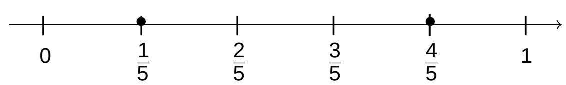
(b) Their denominators are both 5.

So into how many pieces do you need to divide the unit from 0 to 1?_____

How many marks will you need to divide the unit into that many pieces? _____

(c) Divide the unit from 0 to 1 into five equal parts, and label the marks, consecutively, $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$.

(d) Now put points to mark $\frac{1}{5}$ and $\frac{4}{5}$.

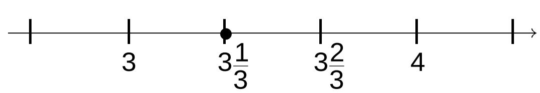


3) The only mixed number to plot is $3\frac{1}{3}$.

(a) Between which two whole numbers is $3\frac{1}{3}$? Remember that a mixed number is a whole number plus a proper fraction, so $3\frac{1}{3} > 3$. Since it is greater than three, but not a whole unit greater, $3\frac{1}{3}$ is between _____and _____.

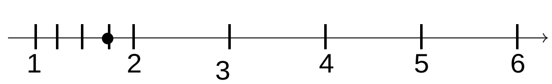
(b) Divide that portion of the number line into _____ equal pieces (thirds) by making ______ marks.

(c) Plot $3\frac{1}{3}$ at the first mark.

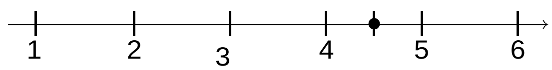


4) Finally, look at the improper fractions $\frac{7}{4}, \frac{9}{2}, \frac{8}{3}$. Locating these points will be easier if you change each of them to a mixed number.

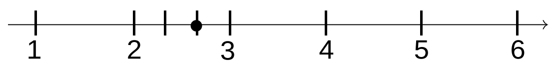
(a) $\frac{7}{4} =$ ______



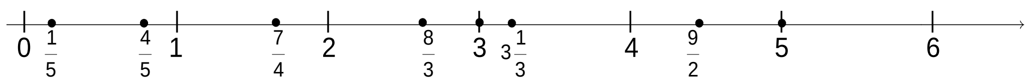
(b) $\frac{9}{2} =$ ______



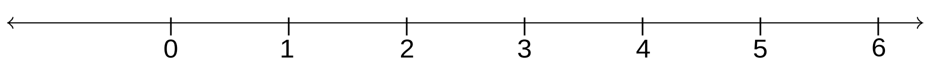
(c) $\frac{8}{3} =$ ______



5) Here is the number line with all the points ($\frac{1}{5}$, $\frac{4}{5}$, 3, $3\frac{1}{3}$, $\frac{7}{4}$, $\frac{9}{2}$, 5, and $\frac{8}{3}$) plotted. Verify that your number line looks the same.



6) Locate and label the fractions $\frac{3}{4}, \frac{4}{3}, \frac{5}{3}, 4\frac{1}{5}, \frac{7}{2}$ on the number line below.



Now let's locate some negative fractions.

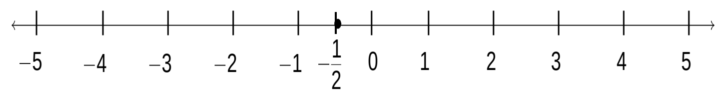
7) We'll locate $-\frac{1}{2}$ first. Remember that negative numbers are opposites of positive numbers, so $-\frac{1}{2}$ is the opposite of $\frac{1}{2}$.

(a) Since $\frac{1}{2}$ is between the two whole numbers _______ and _______, $-\frac{1}{2}$ is between the two integers _______ and ___________.

(b) Into how many pieces do we need to divide the unit between 0 and -1? ________

(c) Divide that portion of the number line into _____ equal pieces (halves) by making ______ marks.

(d) Plot $-\frac{1}{2}$ at the mark.



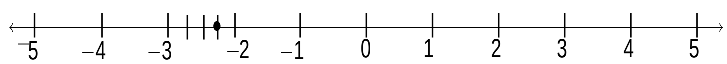
8) Now let's locate $-2\frac{1}{4}$ on a number line.

(a) Think about $2\frac{1}{4}$ first. It is located between the whole numbers ______ and ______.

(b) So $-2\frac{1}{4}$ is between ________ and ________.

(c) Into how many equal pieces do we need to divide that unit? _________

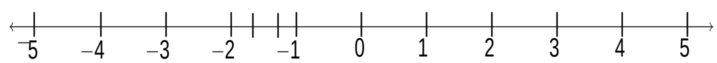
(d) Plot $-2\frac{1}{4}$ at the first mark.



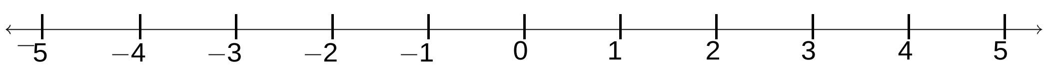
9) Locating $-\frac{5}{3}$ on a number line will be easier if you first change it to a mixed number.

(a) $-\frac{5}{3}$ = ___________. It is between _________ and _________.

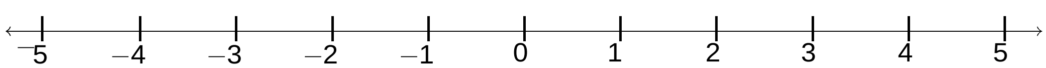
(b) Plot $-\frac{5}{3}$.



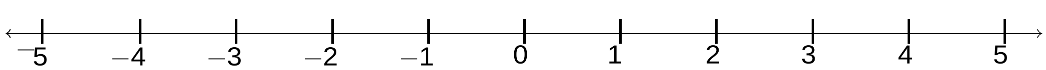
10) Locate and label the fractions $\frac{2}{3}$ and $-\frac{2}{3}$ on the number line below.



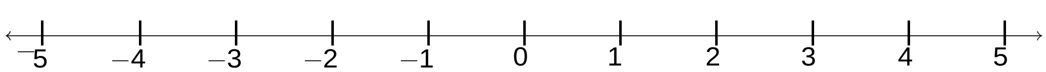
11) Locate and label the fractions $-\frac{9}{2}$ and $-\frac{9}{4}$ on the number line below.



12) Locate and label the fractions $\frac{7}{3}$, $-3\frac{3}{4}$, $3\frac{1}{3}$, and $-\frac{8}{5}$ on the number line below.



13) Locate and label the fractions $\frac{1}{3}$, $-\frac{5}{4}$, $-\frac{7}{4}$, $2\frac{3}{5}$, and $-3\frac{1}{2}$ on the number line below.



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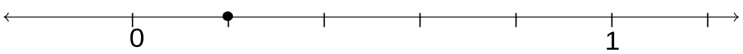
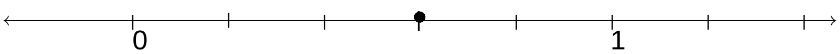
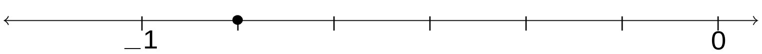
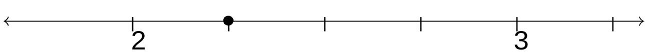
Number Line Part 3 – Extra Practice

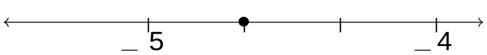
Name the coordinate of each point.

1) 

2) 

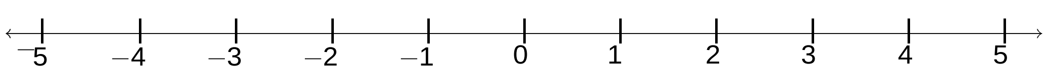
3) 

4) 

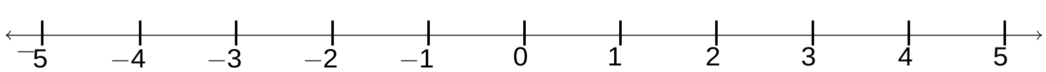
5) 

Locate and label each point on the number line.

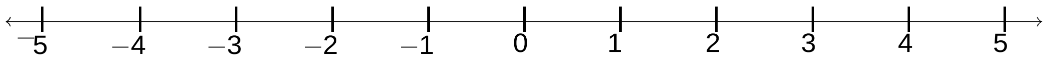
6) (a) $\frac{3}{5}$ (b) $1\frac{2}{3}$ (c) $-\frac{9}{4}$



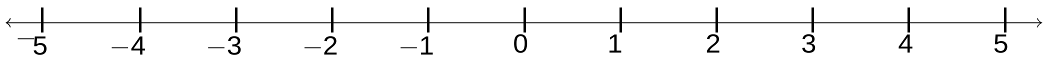
7) (a) $\frac{10}{3}$ (b) $-2\frac{4}{5}$ (c) $-\frac{1}{2}$



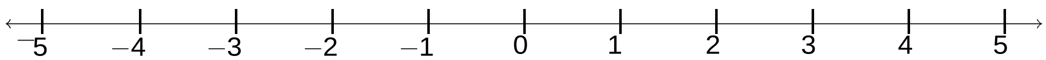
8) (a) $-3\frac{3}{4}$ (b) $\frac{1}{3}$ (c) $\frac{15}{4}$



9) (a) $-\frac{5}{4}$ (b) $4\frac{2}{3}$ (c) $-\frac{4}{5}$



10) (a) $\frac{5}{8}$ (b) $-3\frac{1}{4}$ (c) $-\frac{14}{3}$



You can do more practice locating fractions on the number line at the website
http://www.mathsisfun.com/numbers/fractions-match-frac-line.html.

Manipulative Mathematics **Name**______________________

Multiplication/Factors

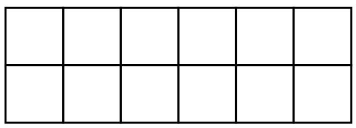
1) Take twelve tiles and form a rectangle.

(a) How many rows does your rectangle have? _____

(b) How many columns? _____

(c) Each rectangle can be called a $\underbrace{\_\_\_\_\_\_}_{\text{number of rows}} \times \underbrace{\_\_\_\_\_\_}_{\text{number of columns}}$ rectangle.

The number of rows and the number of columns are called the **dimensions** of the rectangle.



____rows by ____columns

____×____ rectangle

(d) How many tiles were used to make this rectangle? ____

(e) What is the product of $2 \bullet 6$? ____

(f) Form a 6×2 rectangle. Draw it here:

(g) What do you notice about the 2×6 and the 6×2 rectangles?

A 2×6 rectangle and a 6×2 rectangle are **equivalent**. This means you could rotate the 2×6 rectangle and it would look exactly the same as the 6×2 rectangle.

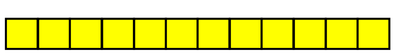
2) Now, create all possible rectangles using 1 tile, 2 tiles, 3 tiles, …, 25 tiles.

(a) Copy each rectangle onto graph paper.

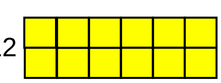
(b) Label each rectangle with the total number of tiles used to form it.

(c) Under the rectangle write its dimensions: $\underbrace{\_\_\_\_\_\_}_{\text{number of rows}} \times \underbrace{\_\_\_\_\_\_}_{\text{number of columns}}$.

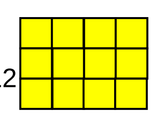
For example, your graph paper would show 3 rectangles for 12 tiles:

12 

1×12

12 

2×6

12 

3×4

(d) Summarize your results in the chart below.

Number of tiles	Dimensions of the rectangles formed
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	$1\times12,\ 2\times6,\ 3\times4$
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	

Use your chart to answer the following questions.

3) Look for all the rectangles in your chart that have 2 rows.

(a) List the dimensions of all the rectangles that have 2 rows.

2×1 , 2×2 ,_____, _____, _____, _____, _____, _____, _____, _____, _____, _____

(b) Now list the total number of tiles you used to form each rectangle you listed in 5.

2, 4, ___, ___, ___, ___, ___, ___, ___, ___, ___, __

These numbers are called the **multiples** of 2.

> **Multiple**
> A number is a multiple of n if it is the product of a counting number and n .

4) How can you use your rectangle chart to find the multiples of 3?

5) List the multiples of three: 3, ___, ___, ___, ___, ___, ___, ___.

6) List the multiples of four: 4, ___, ___, ___, ___, ___.

7) List the multiples of five: 5, ___, ___, ___, ___.

Notice that with 12 tiles, we could form 3 different rectangles, $1\times 12,\ 2\times 6,\text{ and } 3\times 4.$ The numbers 1, 2, 3, 4, 6, and 12 are **factors** of 12, because $1\bullet 12=12,\ 2\bullet 6=12,\text{ and } 3\bullet 4=12.$

Factors

If $a\bullet b=m$, then a and b are factors of m.

8) List all the factors of 15: ___, ___, ___, ___

9) Which number from 1 to 25 has the most factors? ___________

10) Which number of tiles can be used to make the most rectangles? ___________

11) Explain why some numbers can be used to make more rectangles than other numbers.

12) List the numbers for which you could only form one rectangle.

These numbers are called **primes**. A prime number has only two factors, 1 and itself.

Prime

A prime number is a counting number greater than 1, whose only factors are 1 and itself.

13) List all the primes between 2 and 25.

14) What other number relationships do you notice in your rectangle chart?

Manipulative Mathematics **Name______________________**

Multiplication/Factors – Extra Practice

List the first ten multiples of the following numbers.

1) 6: ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

2) 7: ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

3) 8: ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

4) 9: ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

5) 12: ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

The number 16 can be factored $1 \cdot 16$, $2 \cdot 8$, and $4 \cdot 4$, so all the factors of 16 are 1, 2, 4, 8, and 16. Find all the factors of each of the following numbers.

6) 24 ______________________________

7) 30 ______________________________

8) 42 ______________________________

9) 63 ______________________________

10) 135 ______________________________

Manipulative Mathematics **Name**_______________________

Square Numbers

1) Put about 50 color counters on your workspace. We will use the counters to make squares.
   (a) For example,  is a square made from ___ counters. It has ______counters on each side.

   (b) Make as many squares as you can with your counters. Draw a picture of each square that you create and record your results in the table below:

Picture of square							
Total number of counters in the square		4					
Number of counters on each side		2					

2) Can you make a square with exactly 6 counters? ______ Why or why not?

3) Imagine if you had 100 counters
   (a) Could you make a square with exactly 100 counters? ______
   (b) Why or why not?

   (c) How many counters would be on each side of a square made with 100 counters? ___

4) Work with a partner and put all your counters together.
   (a) Create a square that uses more than 50 counters. Draw a sketch of your square.

(b) Create all the squares you can using 50 to 100 counters. Sketch your squares here.

When a number n is multiplied by itself, we write it n^2 and read it 'n squared'. For example,

8^2 is read '8 squared'.

64 is called 'the square of 8'.

Similarly, 121 is the square of 11, because 11^2 is 121.

Square of a number

If $n^2 = m$, then m is the square of n.

5) Complete this table to show the squares of the counting numbers 1 through 15.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n^2								64			121				

The squares of the counting numbers are called **perfect squares**, so the second row of the table shows the first fifteen perfect squares.

6) List the total number of counters you used for each square you made in Exercise 1(b).

7) Do you see a similarity between the table you filled in for Exercise 1 with the pictures of squares and the table you made in Exercise 5 with the squares of the counting numbers 1 through 15?___

(a) Describe how the two tables are alike.

(b) Why do we use the word 'square' for both the symbol in 3^2 and the shape □ ?

Manipulative Mathematics **Name**______________________

Square Numbers – Extra Practice

Identify whether or not each number is a perfect square. If it is a perfect square, write is as the square of a counting number.

Number	Not a perfect square	Yes – perfect square
1) 36	______	______ = _____ ^{2}
2) 50	______	______ = _____ ^{2}
3) 140	______	______ = _____ ^{2}
4) 196	______	______ = _____ ^{2}
5) 221	______	______ = _____ ^{2}
6) 289	______	______ = _____ ^{2}
7) 364	______	______ = _____ ^{2}
8) 625	______	______ = _____ ^{2}
9) 784	______	______ = _____ ^{2}
10) 961	______	______ = _____ ^{2}

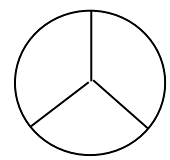
Manipulative Mathematics **Name**______________________

Model Fractions

> **Fraction:**
>
> A fraction is written $\frac{a}{b}$
>
> a is the **numerator** and b is the **denominator**.

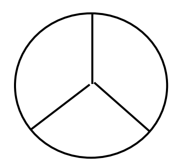
Fractions are a way to represent parts of a whole. The fraction $\frac{1}{3}$ means that one whole has been divided into 3 equal parts and each part is one of the three equal parts.

1) This circle that has been divided into 3 equal parts. Label each part $\frac{1}{3}$.

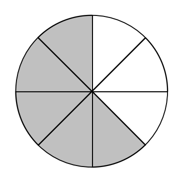


2) What does the fraction $\frac{1}{3}$ represent? This means the whole has been divided into 3 equal parts, and $\frac{1}{3}$ represents two of those three parts.

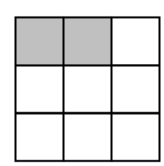
   Shade two out of the three parts of this circle to represent $\frac{2}{3}$.



3) What fraction of this circle is shaded?
   (a) How many parts are shaded? ______
   (b) How many equal parts are there? ______
   (c) The fraction of the circle that is shaded is $\frac{\square}{\square}$.

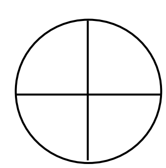


4) What fraction of this square is shaded?
   (a) How many parts are shaded? _____
   (b) How many equal parts are there? _____
   (c) The fraction of the square that is shaded is $\frac{\square}{\square}$



5) To shade – of the circle, shade _____ out of the _____ parts.

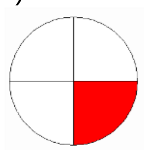
   Shade $\frac{3}{4}$.

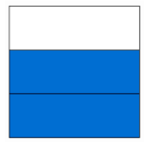


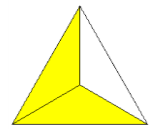
Manipulative Mathematics **Name______________________**

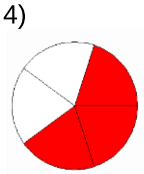
Model Fractions – Extra Practice

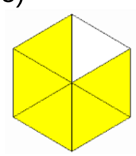
Name the fraction modeled by each figure.

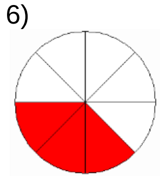
1)

____

2)

____

3)

____

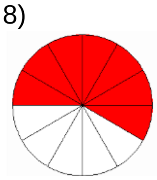
4)

____

5)

____

6)

____

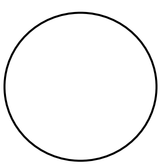
7)

____

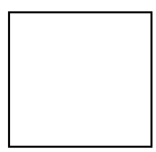
8)

____

Model each fraction.

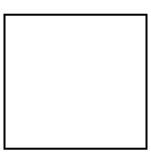
9) $\frac{1}{6}$



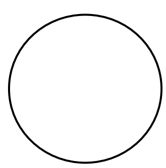
10) $\frac{5}{9}$



11) $\frac{4}{5}$



12) $\frac{7}{8}$



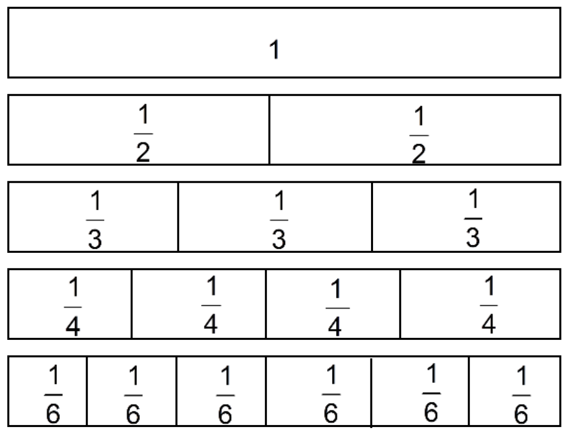
For more practice

- naming fractions, go to: http://nlvm.usu.edu/en/nav/frames_asid_104_g_1_t_1.html?from=topic_t_1.html
- modeling fractions, go to: http://nlvm.usu.edu/en/nav/frames_asid_102_g_2_t_1.html?from=topic_t_1.html

Manipulative Mathematics **Name____________________**

Fractions Equivalent to One

Fractions are often shown as parts of rectangles. Here, the whole is one long rectangle.



Set up your fraction tiles as shown in the diagram above.

1) How many of the $\frac{1}{2}$ tiles does it take to make 1 whole tile?

(a) It takes _____halves to make a whole. (b) Two out of two is 1 whole. $\frac{2}{2}=$_____.

2) How many of the $\frac{1}{3}$ tiles does it take to make 1 whole tile?

(a) It takes _____ thirds to make a whole. (b) Three out of three is 1 whole. $\frac{3}{3}=$_____.

3) How many of the $\frac{1}{4}$ tiles does it take to make 1 whole tile?

(a) It takes _____fourths to make 1 whole. (b) Four out of four is 1 whole. $\frac{4}{4}=$_____.

How many of the $\frac{1}{6}$ tiles does it take to make 1 whole tile?

(a) It takes ______ sixths. (b) Six out of six is 1 whole. $\frac{6}{6}=$_____.

What if the whole was divided into 24 equal parts? We don't have fraction tiles to represent this and it is too many to draw easily, but try to visualize it in your mind.

(a) How many $\frac{1}{24}$'s does it take to make 1? ______ (b) $\frac{\square}{24}=1$

4) Do you see any pattern here? Describe the pattern you see.

Manipulative Mathematics **Name**________________________

Fractions Equivalent to One – Extra Practice

Use fraction tiles to answer these exercises.

You may want to use virtual fraction tiles on the interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html.

1) How many $\frac{1}{5}$'s does it take to make 1? ________________

2) How many $\frac{1}{8}$'s does it take to make 1? ________________

3) How many $\frac{1}{10}$'s does it take to make 1? ________________

4) How many $\frac{1}{13}$'s does it take to make 1? ________________

5) How many $\frac{1}{16}$'s does it take to make 1? ________________

6) How many $\frac{1}{32}$'s does it take to make 1? ________________

7) Fill in each numerator.

(a) $\frac{\square}{9} = 1$ (b) $\frac{\square}{12} = 1$ (c) $\frac{\square}{14} = 1$

8) Fill in each denominator.

(a) $\frac{8}{\square} = 1$ (b) $\frac{11}{\square} = 1$ (c) $\frac{15}{\square} = 1$

9) Fill in the missing part.

(a) $\frac{\square}{7} = 1$ (b) $\frac{20}{20} = \square$ (c) $\frac{25}{\square} = 1$

(d) $\frac{41}{41} = \square$ (e) $\frac{64}{\square} = 1$ (f) $\frac{\square}{100} = 1$

Manipulative Mathematics **Name______________________**

Mixed Numbers and Improper Fractions

1) Use fraction circles to make wholes, if possible, with the following pieces. Draw a sketch to show your result.

(a) 2 halves (b) 6 sixths (c) 4 fourths (d) 5 fifths

2) Use fraction circles to make wholes, if possible, with the following pieces. Draw a sketch to show your result.

(a) 3 halves (b) 5 fourths

(c) 8 fifths (d) 7 thirds

When a fraction has the numerator smaller than the denominator, it is called a **proper** fraction. Its value is less than one. Fractions like $\frac{1}{2}$, $\frac{3}{7}$, and $\frac{11}{18}$ are proper fractions.

A fraction like $\frac{5}{4}$, $\frac{3}{2}$, $\frac{8}{5}$, or $\frac{7}{3}$ is called an **improper** fraction. Its numerator is greater than its denominator. Its value is greater than one.

Proper and Improper Fractions

The fraction $\frac{a}{b}$ is: $(b \neq 0)$

proper if $a < b$ or **improper** if $a \geq b$

3) Write as improper fractions.

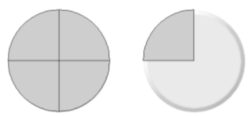
(a) 3 halves_____ (b) 5 fourths _____ (c) 8 fifths_____ (d) 7 thirds_____

4) Look back at your models in Exercise 2 and the improper fractions in Exercise 3. Which improper fraction in Exercise 3 could also be written as $1\frac{1}{4}$? ________

The number $1\frac{1}{4}$ called a **mixed number**; it consists of a whole number and a proper fraction.

> **Mixed Number**
>
> A **mixed number** is written $a\frac{b}{c}$ $\quad c \neq 0$
>
> A mixed number consists of a whole number *a* and a proper fraction $\frac{b}{c}$.

The model shows that $\frac{5}{4}$ has the same value as $1\frac{1}{4}$.



$\frac{5}{4} = 1\frac{1}{4}$

5) Write each improper fraction as a mixed number. You may want to refer to your models in Exercise 2.

(a) $\frac{3}{2}$ _____ (b) $\frac{5}{4}$ _____ (c) $\frac{8}{5}$ _____ (d) $\frac{7}{3}$ _____

6) Rewrite the improper fraction $\frac{11}{6}$ as a mixed number. Use fraction circles to find the result.

(a) Draw a sketch to show your answer. (b) $\frac{11}{6} =$ _____

7) Rewrite the improper fraction $\frac{17}{5}$ as a mixed number. Use fraction circles to find the result.

(a) Draw a sketch to show your answer. (b) $\frac{17}{5} =$ _____

8) Explain how you convert an improper fraction as a mixed number.

9) Rewrite the mixed number $1\frac{2}{3}$ as an improper fraction.

(a) Draw a sketch to show your answer. (b) $1\frac{2}{3} =$ _____

10) Rewrite the mixed number $2\frac{1}{4}$ as an improper fraction.

(a) Draw a sketch to show your answer. (b) $2\frac{1}{4} =$ _____

11) Explain how you convert a mixed number to an improper fraction.

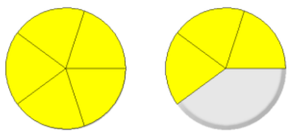
Mixed Numbers and Improper Fractions – Extra Practice

Use 2 sets of fraction circles to do these exercises.

You may want to use the fraction circles on the interactive website http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&from=topic_t_1.html.

Name each improper fraction. Then write each improper fraction as a mixed number.

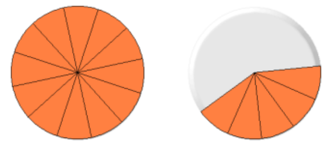
1)



(a) improper fraction ____________

(b) mixed number ____________

2)



(a) improper fraction ____________

(b) mixed number ____________

Draw a figure to model the following improper fractions. Then write each as a mixed number.

Improper fraction	Model	Mixed number
3) $\frac{7}{4}$		$\frac{7}{4} =$
4) $\frac{9}{5}$		$\frac{9}{5} =$
5) $\frac{17}{10}$		$\frac{17}{10} =$
6) $\frac{10}{3}$		$\frac{10}{3} =$

Draw a figure to model the following mixed numbers. Then write each as an improper fraction.

Mixed number	Model	Improper fraction
7) $1\frac{2}{5}$		$1\frac{2}{5} =$
8) $1\frac{1}{6}$		$1\frac{1}{6} =$
9) $1\frac{7}{12}$		$1\frac{7}{12} =$
10) $2\frac{3}{4}$		$2\frac{3}{4} =$

Manipulative Mathematics **Name____________________**

Equivalent Fractions

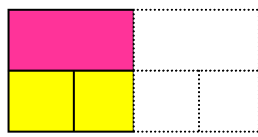
> **Equivalent Fractions**
> Equivalent fractions have the same value.

Use fraction tiles to do the following activity:

1) Take one of the $\frac{1}{2}$ tiles and set it on your workspace.

(a) How many fourths equal one-half?

Take the $\frac{1}{4}$ tiles and place them below the $\frac{1}{2}$ tile.



How many of the $\frac{1}{4}$ tiles exactly cover the $\frac{1}{2}$?______

(b) Since _____ of the $\frac{1}{4}$ tiles cover the $\frac{1}{2}$ tile,

we see $\frac{\square}{4}$ is the same as $\frac{1}{2}$. $\frac{\square}{4} = \frac{1}{2}$

2) How many sixths equal one-half?

(a) How many of the $\frac{1}{6}$ tiles exactly cover the $\frac{1}{2}$ tile?_____

(b) Draw a sketch to show your result.

(c) Since _____ of the $\frac{1}{6}$ tiles cover the $\frac{1}{2}$ tile,

we see $\frac{\square}{6}$ is the same as $\frac{1}{2}$. $\frac{\square}{6} = \frac{1}{2}$

3) How many eighths equal one-half?_____ $\frac{\square}{8} = \frac{1}{2}$

Draw a figure that demonstrates your answer.

4) How many tenths equal one-half?____ $\frac{\square}{10} = \frac{1}{2}$

Draw a figure that demonstrates your answer.

5) How many twelfths equal one-half?____ $\frac{\square}{12} = \frac{1}{2}$

Draw a figure that demonstrates your answer

6) Suppose you had bars marked — .

How many of them would it take to equal one-half?_____ $\frac{\square}{20} = \frac{1}{2}$

Take one of the $\frac{1}{3}$ bars and set it on your workspace.

7) How many sixths equal one-third?_____ $\frac{\square}{6} = \frac{1}{3}$

Draw a figure that demonstrates your answer.

8) How many twelfths equal one-third?_____ $\frac{\square}{12} = \frac{1}{3}$

Draw a figure that demonstrates your answer.

9) Suppose you had tiles marked $\frac{1}{30}$.

How many of them would it take to equal one-third? _____ $\frac{\square}{30} = \frac{1}{3}$

10) How many sixths equal two-thirds? _____ $\frac{\square}{6} = \frac{2}{3}$

Draw a figure that demonstrates your answer.

11) How many eighths equal three-fourths? _____ $\frac{\square}{8} = \frac{3}{4}$

Draw a figure that demonstrates your answer.

12) How many twelfths equal three-fourths? _____ $\frac{\square}{12} = \frac{3}{4}$

Draw a figure that demonstrates your answer.

13) Suppose you had tiles marked $\frac{1}{30}$.

(a) How many of them would it take to equal seven-tenths? _____ $\frac{\square}{30} = \frac{7}{10}$

(b) Explain how you got your answer.

14) Can you use twelfths to make a fraction equivalent to three-fifths? _______
Explain your reasoning.

Manipulative Mathematics **Name____________________**

Equivalent Fractions – Extra Practice

Use fraction tiles to do these exercises. You may want to use virtual fraction tiles on the interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html

1) How many eighths equal one-fourth? _____ $\frac{\square}{8} = \frac{1}{4}$

Draw a figure that demonstrates your answer.

2) How many twelfths equal one-third? _____ $\frac{\square}{12} = \frac{1}{3}$

Draw a figure that demonstrates your answer.

3) How many tenths equal four-fifths? _____ $\frac{\square}{10} = \frac{4}{5}$

Draw a figure that demonstrates your answer.

4) How many sixteenths equal three-fourths? _____ $\frac{\square}{16} = \frac{3}{4}$

Draw a figure that demonstrates your answer.

5) How many fifteenths equal two-thirds? _____ $\frac{\square}{15} = \frac{2}{3}$

Draw a figure that demonstrates your answer.

6) How many fifteenths equal two-fifths? _____ $\frac{\square}{15} = \frac{2}{5}$

Draw a figure that demonstrates your answer.

7) How many twelfths equal six-eighths? _____ $\frac{\square}{12} = \frac{6}{8}$

Draw a figure that demonstrates your answer.

8) How many twelfths equal six-ninths? _____ $\frac{\square}{12} = \frac{6}{9}$

Draw a figure that demonstrates your answer.

Manipulative Mathematics **Name____________________**

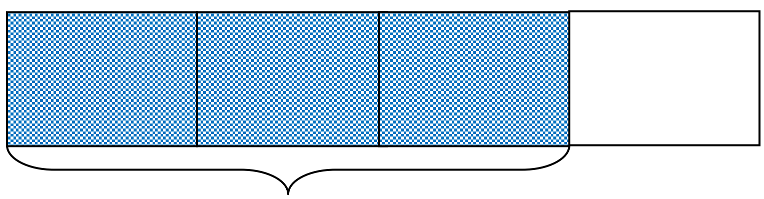
Model Fraction Multiplication

When you multiply fractions, do you need a common denominator? Do you take the reciprocal of one of the fractions? What are you supposed to do and how are you going to remember it? A model may help you understand multiplication of fractions.

1) Model the product $\frac{1}{2} \cdot \frac{3}{4}$.

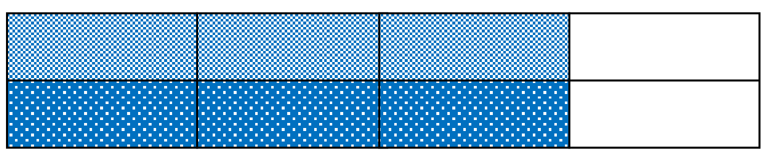
(a) To multiply $\frac{1}{2}$ and $\frac{3}{4}$, let's think "$\frac{1}{2}$ of $\frac{3}{4}$".

(b) First, we draw a rectangle to represent one whole. We divide it vertically into 4 equal parts, and then shade in three of the parts to model $\frac{3}{4}$.



We have shaded in $\frac{3}{4}$ of the rectangle.

(c) Now, we divide the rectangle horizontally into two equal parts to divide the whole into halves. Then we double-shade $\frac{1}{2}$ of what was already shaded.



(d) Into how many equal pieces is the rectangle divided now? ___________

(e) How many of these pieces are double-shaded? _______________

We double-shaded 3 out of the 8 equal pieces, $\frac{3}{8}$ of the rectangle. So $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$.

We showed that

$$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

Notice –

multiplying the numerators $1 \cdot 3 = 3$

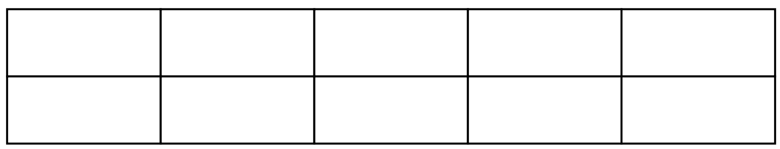
multiplying the denominators $2 \cdot 4 = 8$

2) Model the product $\frac{1}{2} \bullet \frac{3}{5}$.

(a) To multiply $\frac{1}{2}$ and $\frac{3}{5}$, think "$\frac{1}{2}$ of________".

(b) First shade in $\frac{3}{5}$ of the rectangle.

(c) Now double-shade $\frac{1}{2}$ of what was already shaded.



(d) Into how many equal pieces is the rectangle divided now? ___________

(e) How many pieces are double-shaded? _______________

(f) What fraction of the rectangle is double-shaded? ______________________

(g) So $\frac{1}{2}$ of $\frac{3}{5}$ is _______.

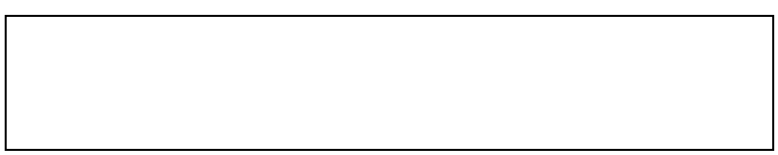
You have shown that

$$\frac{1}{2} \bullet \frac{3}{5} = \frac{3}{10}$$

Notice –
multiplying the numerators $1 \bullet 3 = 3$
multiplying the denominators $2 \bullet 5 = 10$

3) Use a rectangle to model each product. Sketch a diagram to illustrate your model.

(a) $\frac{1}{2} \bullet \frac{1}{3}$



$\frac{1}{2} \bullet \frac{1}{3} =$ ____

(b) $\frac{1}{2} \bullet \frac{1}{4}$

$\frac{1}{2} \bullet \frac{1}{4} =$ ____

(c) $\frac{1}{3} \bullet \frac{1}{4}$

$\frac{1}{3} \cdot \frac{1}{4} =$ ___

(d) $\frac{1}{3} \bullet \frac{2}{3}$

$\frac{1}{3} \bullet \frac{2}{3} =$ ___

(e) $\frac{2}{3} \bullet \frac{4}{5}$

$\frac{2}{3} \bullet \frac{4}{5} =$ ___

4) Look at each of your models and answers in Question 3.

(a) If you multiply numerators and multiply denominators, do you get the same result as you did from the model? ______

(b) Explain in words how to multiply two fractions.

5) The definition of fraction multiplication is given in the box below.

Fraction Multiplication

If a, b, c, and d are numbers where $b \neq 0 \, and \, d \neq 0$, then $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$.

To multiply fractions, multiply the numerators and multiply the denominators.

Use the definition of fraction multiplication to multiply $\frac{5}{12} \bullet \frac{7}{3}$

(a) Identify a, b, c, and d .

(b) Multiply the fractions.

Manipulative Mathematics **Name**______________________

Model Fraction Multiplication – Extra Practice

Use a rectangle to model each multiplication. Sketch your model and write the product.

You can practice using rectangles to model fraction multiplication online at the website http://nlvm.usu.edu/en/nav/frames_asid_194_g_2_t_1.html?from=search.html?qt=multiply+fractions.

1) $\frac{1}{2} \bullet \frac{1}{6}$

2) $\frac{1}{2} \bullet \frac{1}{8}$

3) $\frac{1}{3} \bullet \frac{1}{3}$

4) $\frac{1}{4} \bullet \frac{1}{4}$

5) $\frac{1}{2} \bullet \frac{5}{8}$

6) $\frac{1}{2} \bullet \frac{5}{6}$

Multiply.

7) $\frac{2}{3} \bullet \frac{2}{5}$

8) $\frac{2}{5} \bullet \frac{4}{5}$

9) $\frac{3}{5} \bullet \frac{7}{8}$

10) $\frac{3}{4} \bullet \frac{5}{8}$

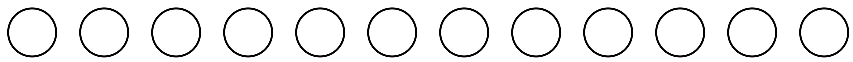
Manipulative Mathematics **Name____________________**

Model Fraction Division

Model Fraction Division

1) Why is $12 \div 3 = 4$? Let's model this with counters.

   (a) How many groups of 3 counters can be made from the 12 shown below?



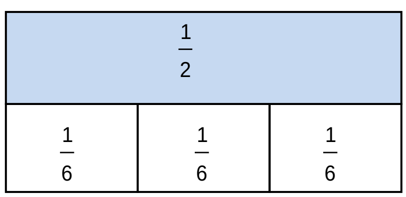
   (b) Draw a circle around each group of 3 counters. How many groups of 3 counters do you have? ____________

   (c) There are _____ groups of 3 counters. In other words, there are _____3s in 12. So, $12 \div 3 =$ _____ .

What about dividing fractions? Get out your fraction tiles and let's see!

2) To model the quotient $\frac{1}{2} \div \frac{1}{6}$ with fraction tiles we want to see how many sixths there are in one-half.

   (a) Line up your half and sixth fraction tiles as shown below.



   (b) How many $\frac{1}{6}$ s are in $\frac{1}{2}$? ________

   (c) $\frac{1}{2} \div \frac{1}{6} =$ ______

3) Model the quotient $\frac{1}{4} \div \frac{1}{8}$ with fraction tiles.

   Use your fourth and eighth fraction tiles to find out how many eighths there are in one fourth.

   (a) Draw a sketch of your result here.

(b) There are ______ $\frac{1}{8}$ s in $\frac{1}{4}$.

(c) So $\frac{1}{4} \div \frac{1}{8} =$ ______

4) Model the quotient $\frac{1}{3} \div \frac{1}{6}$ with fraction tiles

Use your third and sixth fraction tiles to find out how many sixths there are in one third

a) Draw a sketch of your result here.

b) There are ______ $\frac{1}{6}$ s in $\frac{1}{3}$.

c) So $\frac{1}{3} \div \frac{1}{6} =$ ______

5) Model the quotient $\frac{1}{2} \div \frac{1}{8}$ with fraction tiles

Use your half and eighth fraction tiles to find out how many eighths there are in one half.

a) Draw a sketch of your result here.

b) There are ______ $\frac{1}{8}$ s in $\frac{1}{2}$

c) So $\frac{1}{2} \div \frac{1}{8} =$ ______

Model a Whole Number Divided by a Fraction

6) Use fraction bars to model the quotient $2 \div \frac{1}{4}$

(a) How many $\frac{1}{4}$ s are there in 2?

1				1			
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(b) There are ______ $\frac{1}{4}$ s in 2, so $2 \div \frac{1}{4} =$ _____

(c) Let's think of this example another way—in terms of money. We often read $\frac{1}{4}$ as 'one quarter', so you can think of $2 \div \frac{1}{4}$, as asking "how many quarters are there in two dollars?" We know that $1 is 4 quarters, so how many quarters are in $2? _______

(d) So, $2 \div \frac{1}{4} =$ _______ .

7) Use fraction tiles to model the following. Sketch a diagram to illustrate your model.

a) $2 \div \frac{1}{3}$

b) $3 \div \frac{1}{2}$

$2 \div \frac{1}{3} =$ _____

$3 \div \frac{1}{2} =$ _____

Using fraction tiles in exercise 2, we showed that $\frac{1}{2} \div \frac{1}{6} = 3$. Notice that $\frac{1}{2} \bullet \frac{6}{1} = 3$ also. How does $\frac{6}{1}$ relate to $\frac{1}{6}$? They are reciprocals! To divide fractions, we multiply the first fraction by the reciprocal of the second. This leads to the following definition.

Fraction Division

If $a, b, c,$ and d are numbers where $b \neq 0, c \neq 0$ and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$

8) Use the Fraction Division definition above to find the quotient $\frac{5}{7} \div \frac{3}{8}$.

(a) Identify the numbers that correspond to $a, b, c,$ and d.

(b) Divide the fractions.

9) Explain in words how to divide two fractions.

10) Explain in words how to divide a whole number by a fraction.

Manipulative Mathematics **Name**______________________

Model Fraction Division – Extra Practice

Use fraction tiles to model each division. Sketch your model and write the quotient.

You may want to use the fraction tiles shown at the interactive website:
http://www.mathsisfun.com/numbers/fraction-number-line.html.

1) $\frac{1}{2} \div \frac{1}{10}$

2) $\frac{1}{2} \div \frac{1}{12}$

3) $\frac{1}{3} \div \frac{1}{12}$

4) $\frac{1}{4} \div \frac{1}{12}$

5) $\frac{3}{4} \div \frac{1}{8}$

6) $\frac{2}{5} \div \frac{1}{10}$

Divide.

7) $\frac{5}{8} \div \frac{1}{6}$

8) $\frac{5}{6} \div \frac{1}{8}$

9) $\frac{2}{5} \div \frac{1}{2}$

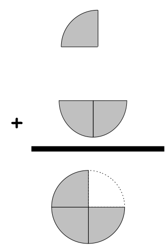
10) $\frac{3}{10} \div \frac{1}{3}$

Manipulative Mathematics **Name____________________**

Model Fraction Addition

How many quarters are pictured above? One quarter plus 2 quarters equals 3 quarters. Quarters? Remember, quarters are really fractions of a dollar; “quarter” is another word for “fourth”. So the picture of the coins shows that $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$.

Let’s use fraction circles to model addition of fractions for the same example, $\frac{1}{4}+\frac{2}{4}$.



Start with one $\frac{1}{4}$ piece. $\frac{1}{4}$

Add two more $\frac{1}{4}$ pieces. $+\frac{2}{4}$

The result is $\frac{3}{4}$. $\frac{3}{4}$

So, $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$.

1) Use fraction circles to model the sum $\frac{3}{8}+\frac{2}{8}$.

(a) Take three $\frac{1}{8}$ pieces. Add two more $\frac{1}{8}$ pieces. How many $\frac{1}{8}$ pieces do you have? ___

(b) Sketch your model here.

(c) You have five eighths. $\frac{3}{8}+\frac{2}{8}=$ _____

2) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{1}{3}+\frac{1}{3}=$ _____ .

(b) $\frac{1}{6}+\frac{4}{6}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

3) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{1}{5}+\frac{3}{5}=$ _____

(b) $\frac{2}{5}+\frac{2}{5}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

4) Use fraction circles to model the following. Sketch a diagram to illustrate your model**.**

(a) $\frac{3}{8}+\frac{4}{8}=$ _____

(b) $\frac{1}{8}+\frac{4}{8}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

5) A common error made by students when adding fractions is to add the numerators and add the denominators (much like we multiply numerators and multiply denominators when multiplying fractions). Use a model to see why this does not work for addition!

(a) Model $\frac{1}{5}+\frac{1}{5}$. Sketch a diagram to illustrate your model.

(b) Did the fifths change to another size piece?_____ Did they change to $\frac{1}{10}$ pieces?_____

(c) $\frac{1}{5}+\frac{1}{5}=$ _____

These examples show that to add the same size fraction pieces—that is, fractions with the same denominator—you just add the number of pieces. So, to add fractions with the same denominator, you add the numerators and place the sum over the common denominator. This leads to the following definition.

Fraction Addition

If $a, b, and\ c$ are numbers where $c \neq 0$, then $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

6) Use the definition of fraction addition in the box above to add $\frac{6}{23} + \frac{8}{23}$.

(a) Identify a, b, and c.

(b) Add the fractions.

.

7) Explain in words how to add two fractions that have the same denominator.

Manipulative Mathematics **Name___________________**

Model Fraction Addition – Extra Practice

Use fraction circles to model each addition. Sketch your model and write the sum.

You may want to use the fraction circles on the interactive website:
http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

1) $\frac{1}{5}+\frac{2}{5}$

2) $\frac{1}{6}+\frac{2}{6}$

3) $\frac{3}{8}+\frac{1}{8}$

4) $\frac{4}{10}+\frac{1}{10}$

5) $\frac{3}{10}+\frac{3}{10}$

6) $\frac{5}{12}+\frac{5}{12}$

7) $\frac{5}{9}+\frac{3}{9}$

8) $\frac{3}{8}+\frac{4}{8}$

9) $\frac{4}{5}+\frac{2}{5}$

10) $\frac{4}{9}+\frac{6}{9}$

11) $\frac{5}{8}+\frac{7}{8}$

12) $\frac{7}{10}+\frac{9}{10}$

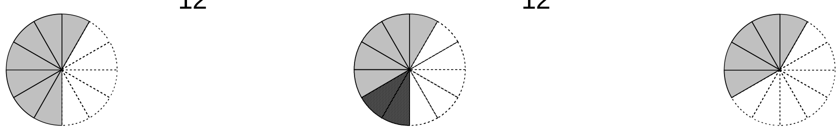
Manipulative Mathematics **Name____________________**

Model Fraction Subtraction

Subtracting two fractions with common denominators works the same as addition of fractions with common denominators. Think of a pizza that was cut into twelve equal slices. Each piece is $\frac{1}{12}$ of the pizza. After dinner there are seven pieces, $\frac{7}{12}$, left in the box. If Leonardo eats 2 of the pieces, $\frac{2}{12}$, how much is left? There would be 5 pieces left, $\frac{5}{12}$. So $\frac{7}{12}-\frac{2}{12}=\frac{5}{12}$.

1) Let's use Fraction Circles to model the same example, $\frac{7}{12}-\frac{2}{12}$.

(a) Start with seven $\frac{1}{12}$ pieces. Take away two $\frac{1}{12}$ pieces.



How many twelfths do you have left?_____

(b) You have five pieces left, $\frac{5}{12}$.

$\frac{7}{12}-\frac{2}{12}=$ _____

2) Use your fraction circles to model the difference $\frac{4}{5}-\frac{1}{5}$.

Start with four $\frac{1}{5}$ pieces. Take away one $\frac{1}{5}$ piece.

(a) How many fifths do you have left?_____

(b) Sketch your model here.

(c) You have ________ fifths left.

$\frac{4}{5}-\frac{1}{5}=$ _____

3) Use fraction circles to model the following. Sketch a diagram to illustrate your model**.**

(a) $\frac{7}{8}-\frac{4}{8}=$ _____

(b) $\frac{5}{6}-\frac{4}{6}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

4) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{3}{4}-\frac{2}{4}=$ ______ (b) $\frac{4}{5}-\frac{2}{5}=$ ______

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

5) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{5}{8}-\frac{2}{8}=$ ______ (b) $\frac{7}{10}-\frac{4}{10}=$ ______

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

These examples show that to subtract the same size fraction pieces—that is, fractions with the same denominator—you just subtract the number of pieces. So, to subtract fractions with the same denominator, you subtract the numerators and place the difference over the common denominator. This leads to the following definition.

Fraction Subtraction

If $a, b, and\ c$ are numbers where $c \neq 0$, then $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$

6) Use the definition of fraction subtraction in the box above to subtract $\frac{11}{17}-\frac{5}{17}$.

(a) Identify a, b, and c .

(b) Subtract the fractions.

7) Explain in words how to subtract two fractions that have the same denominator.

Manipulative Mathematics **Name___________________**

Model Fraction Subtraction – Extra Practice

Use fraction circles to model each subtraction. Sketch your model and write the difference.

You may want to use the fraction circles on the interactive website:
http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

1) $\frac{3}{5}-\frac{1}{5}$

2) $\frac{5}{6}-\frac{1}{6}$

3) $\frac{7}{8}-\frac{1}{8}$

4) $\frac{9}{10}-\frac{1}{10}$

5) $\frac{3}{10}-\frac{3}{10}$

6) $\frac{5}{12}-\frac{5}{12}$

7) $\frac{5}{9}-\frac{3}{9}$

8) $\frac{4}{8}-\frac{3}{8}$

9) $\frac{6}{5}-\frac{2}{5}$

10) $\frac{10}{9}-\frac{4}{9}$

11) $\frac{13}{8}-\frac{5}{8}$

12) $\frac{17}{10}-\frac{7}{10}$

Manipulative Mathematics **Name___________________**

Model Finding the Least Common Denominator

Let's look at coins again. Can you add one quarter and one dime? Well, you could say there are two coins, but that's not very useful. To find the total value of one quarter plus one dime, you change them to the same kind of unit – cents. One quarter equals 25 cents and one dime equals 10 cents, so the sum is 35 cents.

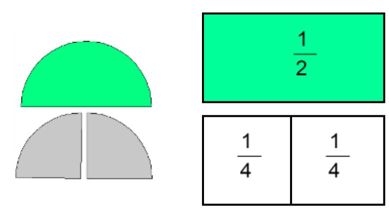
One quarter	+	one dime	25¢ + 10¢
25¢	+	10¢	$\frac{25}{100} + \frac{10}{100}$
	35¢		$\frac{35}{100}$

Similarly, when you add fractions with different denominators you have to convert them to equivalent fractions with a common denominator. With the coins, when we converted to cents, the denominator was 100. 25 cents is $\frac{25}{100}$ and 10 cents is $\frac{10}{100}$ and so we added $\frac{25}{100}+\frac{10}{100}$ to get $\frac{35}{100}$, which is 35 cents.

- Use fraction pieces to find the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$. Take out your set of fraction pieces and place $\frac{1}{2}$ and $\frac{1}{3}$ on your workspace. You need to find a common fraction piece that can be used to cover both $\frac{1}{2}$ and $\frac{1}{3}$ **exactly**.

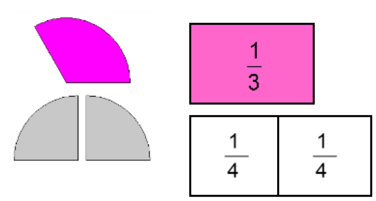
1) Try the $\frac{1}{4}$ pieces.

fraction circles fraction tiles



(a) Can you cover the $\frac{1}{2}$ piece exactly with $\frac{1}{4}$ pieces? _____

(b) How many $\frac{1}{4}$ pieces cover the $\frac{1}{2}$ piece? _____



(c) Can you cover the $\frac{1}{3}$ piece exactly with $\frac{1}{4}$ pieces? _____

(d) How many $\frac{1}{4}$ pieces cover the $\frac{1}{3}$ piece?_____

(e) Sketch your results here.

2) Try the $\frac{1}{5}$ pieces.

(a) Can you cover the $\frac{1}{2}$ piece exactly with $\frac{1}{5}$ pieces? _____

(b) How many $\frac{1}{5}$ pieces cover the $\frac{1}{2}$ piece?___

(c) Can you cover the $\frac{1}{3}$ piece exactly with $\frac{1}{5}$ pieces?_____.

(d) How many $\frac{1}{5}$ pieces cover the $\frac{1}{3}$ piece?___

(e) Sketch your results here.

3) Try the $\frac{1}{6}$ pieces.

(a) Can you cover the $\frac{1}{2}$ piece exactly with $\frac{1}{6}$ pieces? _____

(b) How many $\frac{1}{6}$ pieces cover the $\frac{1}{2}$ piece? ___

(c) Can you cover the $\frac{1}{3}$ piece exactly with $\frac{1}{6}$ pieces? _____.

(d) How many $\frac{1}{6}$ pieces cover the $\frac{1}{3}$ piece? ___

(e) Sketch your results here.

4) You have shown that:

(a) 3 of the $\frac{1}{6}$ pieces exactly cover the $\frac{1}{2}$ piece. $\frac{1}{2} = \frac{\ }{6}$

(b) 2 of the $\frac{1}{6}$ pieces exactly cover the $\frac{1}{3}$ piece. $\frac{1}{3} = \frac{\ }{6}$

The smallest denominator of a fraction piece that can be used to cover both fractions exactly is the **least common denominator** (LCD) of the two fractions. The smallest denominator of a fraction piece that can be used to cover both $\frac{1}{2}$ and $\frac{1}{3}$ is 6. So, you have found that the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$ is 6.

- Use fraction pieces to find the least common denominator of $\frac{1}{4}$ and $\frac{1}{6}$. Place $\frac{1}{4}$ and $\frac{1}{6}$ on your workspace. Find a common fraction piece that can be used to cover both $\frac{1}{4}$ and $\frac{1}{6}$ exactly.

5) Sketch your results here.

6) You have shown that:

(a) _____ of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{4}$ piece. $\frac{1}{4} = —$

(b) _____of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{6}$ piece. $\frac{1}{6} = —$

(c) Both fractions can be written with denominator_____, so _____ is their common denominator.

- Use fraction pieces to find the least common denominator of $\frac{1}{4}$ and $\frac{1}{3}$. Find a common fraction piece that can be used to cover both $\frac{1}{4}$ and $\frac{1}{3}$ exactly.

7) Sketch your results here.

8) You have shown that:

(a) _____ of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{4}$ piece . $\frac{1}{4} = —$

(b) _____of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{3}$ piece. $\frac{1}{3} = —$

(c) Both fractions can be written with denominator_____, so ____ is their common denominator.

- Use fraction pieces to find the least common denominator of $\frac{1}{2}$ and $\frac{1}{5}$. Find a common fraction piece that can be used to cover both $\frac{1}{2}$ and $\frac{1}{5}$ exactly.

9) Sketch your results here.

10) You have shown that:

(a) _____ of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{2}$ piece. $\frac{1}{2} = \frac{}{}$

(b) _____of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{5}$ piece. $\frac{1}{5} = \frac{}{}$

(c) Both fractions can be written with denominator_____, so ____ is their common denominator.

Manipulative Mathematics **Name____________________**

Model Finding the Least Common Denominator – Extra Practice

Use fraction tiles or fraction circles to find the least common denominator (LCD) of each pair of fractions, and to re-write each fraction with the LCD. Sketch your model.

You may want to use the fraction tiles on the interactive website: http://www.mathsisfun.com/numbers/fraction-number-line.html or the fraction circles at http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html to work these exercises.

1) $\frac{1}{3}$ and $\frac{1}{6}$

(a) LCD = _______

(b) $\frac{1}{3}$ = _________

(c) $\frac{1}{6}$ = _________

(d) sketch your model.

2) $\frac{1}{2}$ and $\frac{1}{8}$

(a) LCD = _______

(b) $\frac{1}{2}$ = _________

(c) $\frac{1}{8}$ = _________

(d) sketch your model.

3) $\frac{2}{3}$ and $\frac{1}{12}$

(a) LCD = _______

(b) $\frac{2}{3}$ = _________

(c) $\frac{1}{12}$ = _________

(d) sketch your model.

4) $\frac{3}{4}$ and $\frac{1}{12}$

(a) LCD = _______

(b) $\frac{3}{4}$ = _________

(c) $\frac{1}{12}$ = _________

(d) sketch your model.

5) $\frac{3}{8}$ and $\frac{3}{4}$

(a) LCD = _______

(b) $\frac{3}{8}$ = _________

(c) $\frac{3}{4}$ = _________

(d) sketch your model.

6) $\frac{5}{6}$ and $\frac{2}{3}$

(a) LCD = _______

(b) $\frac{5}{6}$ = _________

(c) $\frac{1}{2}$ = _________

(d) sketch your model.

7) $\frac{1}{2}$ and $\frac{2}{5}$

(a) LCD = ________

(b) $\frac{1}{2}$ = __________

(c) $\frac{2}{5}$ = __________

(d) sketch your model.

8) $\frac{1}{3}$ and $\frac{3}{4}$

(a) LCD = ________

(b) $\frac{1}{3}$ = __________

(c) $\frac{3}{4}$ = __________

(d) sketch your model.

9) $\frac{5}{12}$ and $\frac{2}{3}$

(a) LCD = ________

(b) $\frac{5}{12}$ = __________

(c) $\frac{2}{3}$ = __________

(d) sketch your model.

10) $\frac{3}{4}$ and $\frac{5}{6}$

(a) LCD = ________

(b) $\frac{3}{4}$ = __________

(c) $\frac{5}{6}$ = __________

(d) sketch your model.

Manipulative Mathematics **Name______________________**

Addition of Signed Numbers

We are going to model signed numbers with two-color counters. One white counter, ○, will represent one positive unit. One red counter, ●, will represent one negative unit.

When we have one positive and one negative together, ○ ● we call it a 'neutral pair'. The value of a neutral pair is zero.

1) We'll start by modeling $5+3$, the sum of 5 and 3.
   (a) Start with 5 positives. ○ ○ ○ ○ ○
   (b) Add 3 positives. Put counters of the same color in the same row. ○ ○ ○ ○ ○ ○ ○ ○
   (c) How many counters are there? _________ positives

   $5+3=8$

2) Now we'll model $-5+(-3)$, the sum of negative 5 and negative 3.
   (a) Start with 5 negatives. ● ● ● ● ●
   (b) Add 3 negatives. ● ● ● ● ● ● ● ●
   (c) How many counters are there? _________ negatives

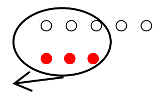
   $-5+ -3 =-8$

3) What about adding numbers with different signs? Let's model $-5+3$, the sum of negative 5 and 3.
   (a) Start with 5 negatives. ● ● ● ● ●
   (b) Add 3 positives. Since they are a different color, line them up under the red counters.
   ● ● ● ● ●
   ○ ○ ○
   (c) Are there any neutral pairs? ______
   Remove the neutral pairs.
   ● ● ● ● ●
   ○ ○ ○
   (d) How many are left? ● ● _______ negatives

   $-5+3=-2$

4) The fourth case is the sum of a positive and a negative. We'll model $5+(-3)$, the sum of 5 and negative 3.
   (a) Start with 5 positives. ○ ○ ○ ○ ○
   (b) Add 3 negatives.
   ○ ○ ○ ○ ○
   ● ● ●

(c) Remove the neutral pairs.



(d) How many are left? ○ ○ 2 ______________

$5 + -3 = 2$

Use your counters to model each sum. Draw a sketch of your model.

5) $4+2$ 6) $-5+(-5)$ 7) $-1+4$ 8) $2+(-4)$

9) $8+(-4)$ 10) $7+(-3)$ 11) $-2+(-3)$ 12) $-5+7$

13) $-2+(-1)$ 14) $-3+3$ 15) $7+(-2)$ 16) $-4+2$

17) Do you notice a pattern? Explain in words how to add:

(a) $-8+(-10)$

(b) $25 + -5$

18) Without using counters, try to find these sums.

(a) $35+29$ (b) $-57+(-43)$

(c) $78+(-74)$ (d) $-64+31$

Manipulative Mathematics **Name**______________________

Addition of Signed Numbers – Extra Practice

Use two-color counters to model each addition.

You can find virtual counters on the website: http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html?from=topic_t_1.html. If you use the website, click on 'User' at the bottom of the workspace so that you can enter the numbers in each exercise.

Sketch the model for each addition and find the sum.

1) $5+2$

2) $-5+(-2)$

3) $-5+2$

4) $5+(-2)$

5) $6+(-6)$

6) $3+(-1)$

7) $-4+(-5)$

8) $-6+8$

9) $-3+(-7)$

10) $-2+2$

11) $4+(-8)$

12) $-4+9$

Manipulative Mathematics **Name______________________**

Subtraction of Signed Numbers

We are going to model signed numbers with two-color counters. One white counter, ○, will represent one positive unit. One red counter, ●, will represent one negative unit.

When we have one positive and one negative together, ○ ● we call it a 'neutral pair'. The value of a neutral pair is zero.

1) We'll start by modeling $5-3$, the difference of 5 and 3.
   (a) Start with 5 positives. ○ ○ ○ ○ ○
   (b) Take away 3 positives. ○ ○ ○ ○ ○
   (c) How many counters are left? ________ positives

   $5-3=2$

2) Now we'll model $-5-(-3)$, the difference of negative 5 and negative 3.
   (a) Start with 5 negatives. ● ● ● ● ●
   (b) Take away 3 negatives. ● ● ● ● ●
   (c) How many counters are left? ● ● _______ negatives

   $-5- \;\; -3 = -2$

3) What about subtracting numbers with different signs? Let's model $-5-3$, the difference of negative 5 and 3.
   (a) Start with 5 negatives. ● ● ● ● ●
   (b) We want to take away 3 positives. Do we have any positives to take away? __________
   (c) We can add 3 neutral pairs to get the 3 positives. ● ● ● ● ● ● ● ● / ○ ○ ○
   (d) Now take away 3 positives. ● ● ● ● ● ● ● ● / ○ ○ ○
   (e) How many counters are left? ● ● ● ● ● ● ● ●

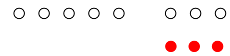
   8 ____________

   $-5-3=-8$

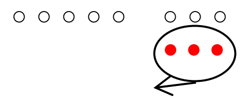
4) The fourth case is the sum of a positive and a negative. We'll model $5-(-3)$, the difference of 5 and negative 3.
   (a) Start with 5 positives. ○ ○ ○ ○ ○

(b) We want to take away 3 negatives. Do we have any negatives to take away? ____________

(c) But we can add 3 neutral pairs to get the 3 negatives.



(d) Take away 3 negatives.



(e) How many counters are left?

8 ______________

$5 - -3 = 8$

Use your counters to model each difference. Draw a sketch of your model.

5) $7-2$

6) $6-(-4)$

7) $-1-4$

8) $-3-(-2)$

9) $3-(-4)$

10) $-5-(-1)$

11) $8-6$

12) $-7-3$

13) $-4-(-4)$

14) $-3-3$

15) $1-5$

16) $-2- -6$

17) Do you notice a pattern? Explain in words how to subtract:

(a) $-8-(-2)$

(b) $-10-5$

18) Without using counters, try to find these differences.

(a) $35-29$

(b) $-57-(-43)$

(c) $78-(-74)$

(d) $-64-31$

Manipulative Mathematics **Name_______________________**

Subtraction of Signed Numbers – Extra Practice

Use two-color counters to model each subtraction.

You can find virtual counters on the website http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html?from=topic_t_1.html. If you use the website, click on 'User' at the bottom of the workspace so that you can enter the numbers in each exercise.

Sketch the model for each subtraction and find the difference.

1) $7-2$

2) $7-(-2)$

3) $-7-2$

4) $-7-(-2)$

5) $6-(-5)$

6) $-4-(-1)$

7) $-8-8$

8) $9-5$

9) $-3-(-3)$

10) $5-4$

11) $-2- -6$

12) $4-10$

Manipulative Mathematics **Name**____________________

Multiples

Multiple of a Number
A number is a multiple of n if it is the product of a counting number and n.

1) **Multiples of 2**

(a) This table lists the counting numbers from 1 to 50. Highlight all the multiples of 2.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

(b) Now look at all the numbers that you highlighted. Describe a pattern you notice.

(c) Create a rule you could use to determine if a number larger than 50 is a multiple of 2.

(d) Use your rule to decide if 497 is a multiple of 2.

(e) Is 846 a multiple of 2?

2) **Multiples of 5**

(a) This table lists the counting numbers from 1 to 50. Highlight all the multiples of 5.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

(b) Now look at all the numbers that you highlighted. Describe a pattern you notice.

(c) Create a rule you could use to determine if a number larger than 50 is a multiple of 5.

(d) Use your rule to decide if 741 is a multiple of 5.

(e) Is 940 a multiple of 5?

3) **Multiples of 10**

(a) The table lists the counting numbers from 1 to 50. Highlight all the multiples of 10.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

(b) Now look at all the numbers that you highlighted. Describe a pattern you notice.

(c) Create a rule you could use to determine if a number larger than 50 is a multiple of 10.

(d) Use your rule to decide if 690 is a multiple of 10.

(e) Is 875 a multiple of 10?

4) **Multiples of 3**

(a) The table lists the counting numbers from 1 to 50. Highlight all the multiples of 3.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

(b) List the multiples of 3.

(c) Under each multiple of 3, find the sum of the digits of that number. For example, 42 is a multiple of 3, and $4+2=6$. What do you notice about all the multiples of 3?

$$\begin{array}{rccccccc} & 3, & 6, & 9, & 12, & 15, & 18, \ldots\ldots\ldots\ldots, & 42, \ldots \\ \textit{sum of digits} & 3 & 6 & 9 & \underbrace{1+2}_{3} & \underbrace{1+5}_{6} & \underbrace{1+8}_{9} & \underbrace{4+2}_{6} \end{array}$$

(d) Use these results to create a rule to determine if a number is a multiple of 3.

(e) Use your rule to decide if 375 is a multiple of 3.

(f) Is 1488 a multiple of 3?

Manipulative Mathematics **Name**______________________

Multiples – Extra Practice

1) State a rule you can use to determine if a number is a multiple of:

(a) 2 ____

(b) 3 ____

(c) 5 ____

(d) 10 ____

For each number, determine if it is a multiple of 2, 3, 5, and/or 10, and indicate your answers by writing 'yes' or 'no' in the spaces below.

		(a) multiple of 2	(b) multiple of 3	(c) multiple of 5	(d) multiple of 10
2)	130	(a)_________	(b) _________	(c) _________	(d) _________
3)	165	(a) _________	(b) _________	(c) _________	(d) _________
4)	225	(a) _________	(b) _________	(c) _________	(d) _________
5)	234	(a) _________	(b) _________	(c) _________	(d) _________
6)	255	(a) _________	(b) _________	(c) _________	(d) _________
7)	270	(a) _________	(b) _________	(c) _________	(d) _________
8)	625	(a) _________	(b) _________	(c) _________	(d) _________
9)	1155	(a) _________	(b) _________	(c) _________	(d) _________
10)	1650	(a) _________	(b) _________	(c) _________	(d) ________

Manipulative Mathematics **Name**________________________

Prime Numbers

> **Prime Number**
> A prime number is a counting number greater than 1, whose only factors are one and itself.
> A counting number that is not prime is **composite**.

1) Use this table to find the primes less than 50. Remember a prime number is a number whose only factors are 1 and itself. The number 1 is not considered prime, so the smallest prime number is 2.

   (a) On the table, circle 2 and then cross out all the multiples of 2. All multiples of 2, greater than 2, have two as a factor and so are not prime.

   (b) Next, circle 3 and then cross out all the multiples of 3. All multiples of 3, greater that 3, have three as a factor and so are not prime.

   (c) Go to the next number that has not been crossed out. Circle it—it is prime—and then cross out all its multiples.

   (d) Continue this routine until all the numbers in the table have been crossed out or circled.

~~1~~	(2)	3	~~4~~	5	~~6~~	7	~~8~~	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

2) The numbers that have been crossed out are not prime. Counting numbers that are not prime are called ______________________________.

3) The circled numbers are prime. List the primes less than 50.

4) What are the only factors of each prime you listed?

5) State one fact you notice about the primes.

Manipulative Mathematics **Name**______________________

Prime Numbers – Extra Practice

1) Use this table to find the primes less than 100. Remember a prime number is a number whose only factors are 1 and itself. The number 1 is not considered prime, so the smallest prime number is 2.

   (a) On the table, circle 2 and then cross out all the multiples of 2. All multiples of 2, greater than 2, have two as a factor and so are not prime.

   (b) Next, circle 3 and then cross out all the multiples of 3. All multiples of 3, greater than 3, have three as a factor and so are not prime.

   (c) Go to the next number that has not been crossed out. Circle it—-it is prime—and then cross out all its multiples.

   (d) Continue this routine until all the numbers in the table have been crossed out or circled.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

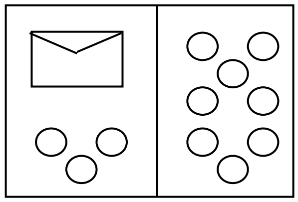
2) The circled numbers are prime. List the primes less than 100.

For more practice online, you can use the hundreds chart at http://nlvm.usu.edu/en/nav/frames_asid_158_g_3_t_1.html?open=instructions&hidepanel=true&from=topic_t_1.html. Display 10 rows to show the numbers 1 to 100 and click 'Remove Multiples' at the bottom of the workspace to remove (instead of crossing out) the multiples of each prime.

Manipulative Mathematics **Name_____________________**

Subtraction Property of Equality

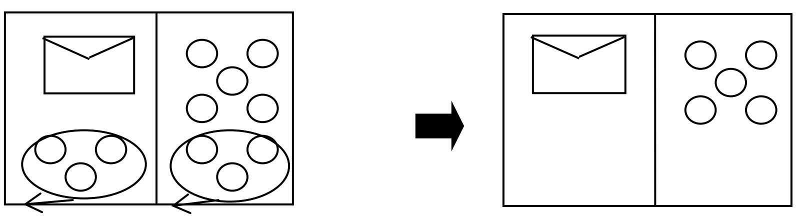
1) You are going to solve a puzzle. Use your envelopes and counters to recreate the picture below on your workspace. Both sides have the same number of counters, but some counters are “hidden” in the envelope. The goal is to discover how many counters are in the envelope.



(a) How many counters are in the envelope? _____counters are in the envelope.

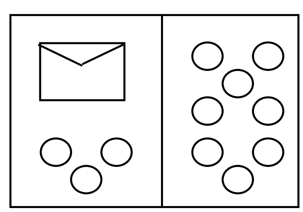
(b) What are you thinking? What steps are you taking in your mind to figure out how many counters are in the envelope? List the steps here.

Perhaps you are thinking- the 3 counters at the bottom left can be matched with 3 on the right. Then I can take them away from both sides. That leaves five on the right-so there must be 5 counters in the envelope. Try this with your envelope and counters.



(c) Each side of the workspace models an expression and the line in the middle represents the equal sign, so we can write an algebraic equation from this model.

What algebraic equation is modeled by this picture?



_____________ = _______

Let's write algebraically the steps we took to discover how many counters were in the envelope:

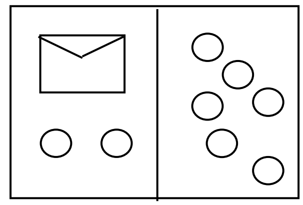
$$x + 3 = 8$$

We took away three from each side. $x + 3 - \_\_\_ = 8 - \_\_\_$

And then we had ____ left. $x = 5$

(d) Check: $\_\_\_ + 3 = 8$
Five in the envelope plus three more equals eight!

2) Let's try this again! How many counters are in the envelope?



Use your envelope and counters to recreate this picture. Now, move the counters to find out how many counters are in the envelope.

(a) List the steps you took to find out how many counters were in the envelope.

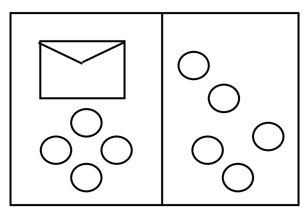
(b) What algebraic equation is modeled by this picture? $x + \_\_\_\_\_ = \_\_\_\_\_$

(c) We need to take away ______from each side. $x + 2 - \_\_\_\_ = 6 - \_\_\_\_$

(d) There are _____counters in the envelope! $x = \_\_\_\_$

(e) Check: $\_\_\_ + 2 = 6$
Four in the envelope plus two more does equal six!

3) How many counters are in this envelope?



Use your envelope and counters to recreate this picture. Move the counters to discover how many counters are in the envelope.

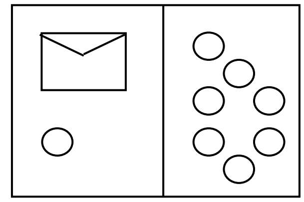
(a) Write the algebraic equation that is modeled by this picture. $x + \_\_\_\_\_ = \_\_\_\_\_$

(b) Take away _____ from each side. $x + 4 - \_\_\_\_ = 5 - \_\_\_\_$

(c) There are _____counters in the envelope! $x = \_\_\_\_$

(d) Check: $\_\_\_ + 4 = 5$

4) How many counters are in this envelope?



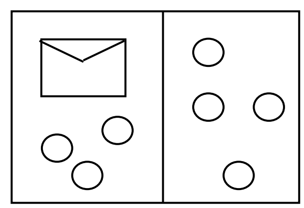
Use your envelope and counters to recreate this picture. Move the counters to find the number of counters in the envelope.

(a) Write the equation modeled by the envelope and counters. _________=__________

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

5) How many counters are in this envelope?



Use your envelopes and counters to recreate this picture. Move the counters as needed to find the number of counters in the envelope.

(a) Write the equation modeled by the envelope and counters. _________=__________

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

6) Model a similar equation for your partner. Have your partner figure out how many counters are in the envelope.
   (a) Sketch a picture of your model.

   (b) Show the algebra steps your partner took to find the number of counters in the envelope.

7) Have your partner model a similar equation for you. Figure out how many counters are in the envelope.
   (a) Sketch a picture of the model.

   (b) Show the algebra steps you took to find the number of counters in the envelope.

With these puzzles we have modeled a method for solving one kind of equation. To solve each equation, we used the Subtraction Property of Equality.

The Subtraction Property of Equality:
For any real numbers a, b, and c,
if $a = b$, then $a - c = b - c$.

When you subtract the same quantity from both sides of an equation, you still have equality!

Manipulative Mathematics **Name______________________**

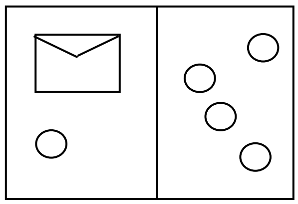
Subtraction Property of Equality – Extra Practice

#1-6: For each figure:

(a) Write the equation modeled by the envelopes and counters.

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

1)

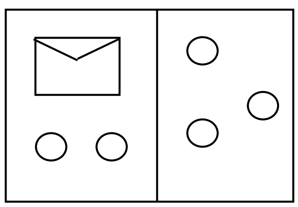


(a) Equation ___________=__________

(b) Solution

Words	Algebra

2)

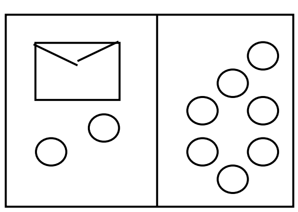


(a) Equation ___________=__________

(b) Solution

Words	Algebra

3)

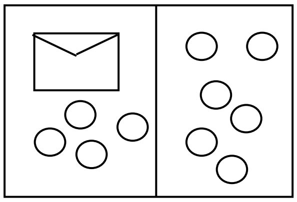


(a) Equation ___________=__________

(b) Solution

Words	Algebra

4)

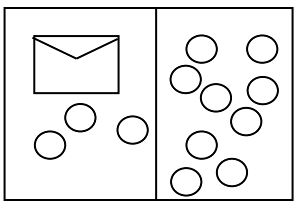


(a) Equation ___________=__________

(b) Solution

Words	Algebra

5)

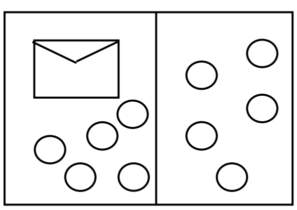


(a) Equation ___________=__________

(b) Solution

Words	Algebra

6)



(a) Equation ___________=__________

(b) Solution

Words	Algebra

#7-18: Solve each equation using the Subtraction Property of Equality.

7)

$$x+3=5$$
$$x+3-\_\_\_=5-\_\_\_$$
$$x=\_\_\_$$

8)

$$x+2=10$$
$$x+2-\_\_\_=10-\_\_\_$$
$$x=\_\_\_$$

9)

$$x+9=17$$
$$x+9-\_\_\_=17-\_\_\_$$
$$x=\_\_\_$$

10)

$$x+14=23$$
$$x+14-\_\_\_=23-\_\_\_$$
$$x=\_\_\_$$

11)

$$x+36=51$$
$$x+36-\_\_\_=51-\_\_\_$$
$$x=\_\_\_$$

12)

$$x+75=102$$
$$x+75-\_\_\_=102-\_\_\_$$
$$x=\_\_\_$$

13) $x+18=33$

14) $y+29=100$

15) $u+72=241$

16) $v+325=465$

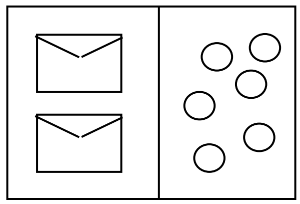
17) $m+593=902$

18) $n+762=2014$

Manipulative Mathematics **Name___________________**

Division Property of Equality

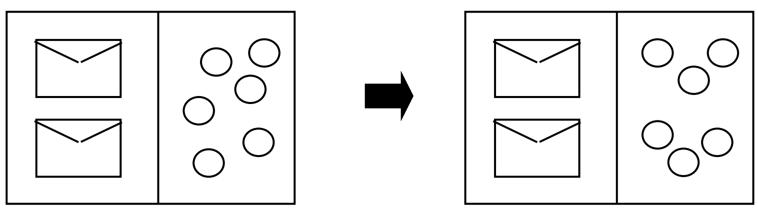
1) You are going to solve a puzzle. Use your envelopes and counters to recreate the picture below on your workspace. Both sides have the same total number of counters, but some counters are "hidden" in the envelopes. Both envelopes contain the same number of counters. The goal is to discover how many counters are in each envelope.



(a) How many counters are in each envelope? _____counters are in each envelope.

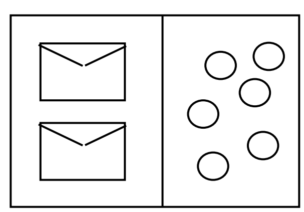
(b) What are you thinking? What steps are you taking in your mind to figure out how many counters are in each envelope? List the steps here.

Perhaps you are thinking that you have to separate the counters on the right side into 2 groups, because there are 2 envelopes. So 6 counters divided into 2 groups means there must be 3 counters in each envelope. Try this with your envelopes and counters.



(c) Each side of the workspace models an expression and the line in the middle represents the equal sign, so we can write an algebraic equation from this model.

What algebraic equation is modeled by this picture?



___________ = ______

(d) Let's write algebraically the steps we took to discover how many counters were in the envelope:

$$2x = 6$$

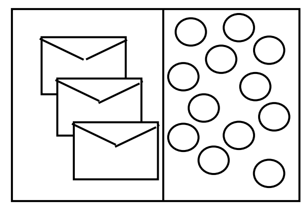
We divided both sides of the equation by _____, $\frac{2x}{\square} = \frac{6}{\square}$

So we have _____ in each envelope. $x = 3$

(e) Check: $2 \bullet \_\_\_ = 6$ Three counters in each of two envelopes equals six!

2) Here's another puzzle. How many counters are in each envelope?

Use your envelopes and counters to recreate this picture. Now, move the counters to find out how many counters are in each envelope.



(a) List the steps you took to find out how many counters are in each envelope.

(b) What algebraic equation is modeled by this picture? $\_\_\_x = \_\_\_\_\_$

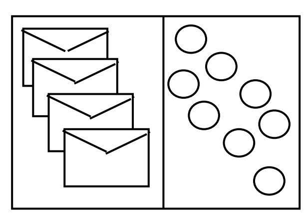
(c) We need to divide the counters into _____groups.

(d) Divide each side by _____. $\frac{3x}{\square} = \frac{12}{\square}$

(e) There are _____counters in each envelope! $x = \_\_\_\_$

(f) Check: $3 \bullet \_\_\_ = 12$

3) How many counters are in each envelope?
Use your envelopes and counters to recreate this picture. Move the counters to discover how many counters are in each envelope.



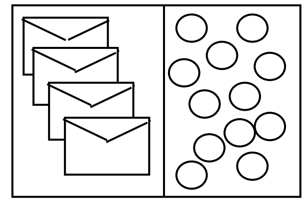
(a) Write the algebraic equation that would match this situation. $\_\_\_x = \_\_\_\_\_$

(b) Divide each side by_______. $\frac{4x}{\square} = \frac{8}{\square}$

(c) There are _____counters in each envelope! $x = \_\_\_\_$

(d) Check: $4 \bullet \_\_\_ = 8$

4) How many counters are in each envelope?
Use your envelopes and counters to recreate this picture.
Move the counters to find the number of counters in the envelope.

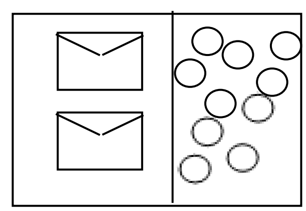


(a) Write the equation modeled by the envelopes and counters. ___x = _____

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

5) How many counters are in each envelope?
Use your envelopes and counters to recreate this picture.
Move the counters as needed to find the number of counters in the envelope.



(a) Write the equation modeled by the envelopes and counters. ___x = _____

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

6) Model a similar equation for your partner. Have your partner figure out how many counters are in each envelope.
   (a) Sketch a picture of your model.

   (b) Show the algebra steps your partner took to find the number of counters in each envelope.

7) Have your partner model a similar equation for you. Figure out how many counters are in each envelope.
   (a) Sketch a picture of the model.

   (b) Show the algebra steps you took to find the number of counters in each envelope.

With these puzzles we have modeled a method for solving one kind of equation. To solve each equation, we used the Division Property of Equality.

The Division Property of Equality

For any real numbers a, b, c, and $c \neq 0$,

if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

When you divide both sides of an equation by any non-zero number, you still have equality!

 Name________________________

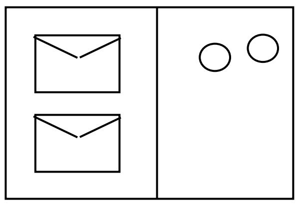
Division Property of Equality – Extra Practice

#1-6: For each figure:

(a) write the equation modeled by the envelopes and counters.

(b) show the steps you take, in words and algebra, to find the number of counters in each envelope.

1)

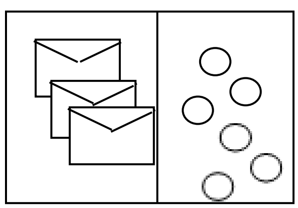


(a) Equation ___x = _____

(b) Solution

Words	Algebra

2)

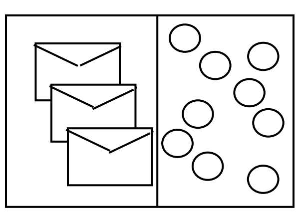


(a) Equation ___x = _____

(b) Solution

Words	Algebra

3)

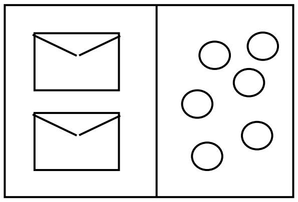


(a) Equation ___x = _____

(b) Solution

Words	Algebra

4)

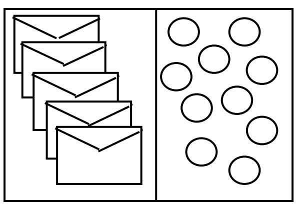


(a) Equation ___ $x =$ ______

(b) Solution

Words	Algebra

5)

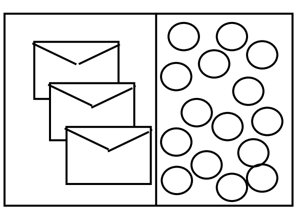


(a) Equation ___ $x =$ ______

(b) Solution

Words	Algebra

6)



(a) Equation ___ $x =$ ______

(b) Solution

Words	Algebra

#7-18: Solve each equation using the Division Property of Equality.

7)
$2x = 16$

$\frac{2x}{\square} = \frac{16}{\square}$

$x = \_\_\_$

8)
$4x = 16$

$\frac{4x}{\square} = \frac{16}{\square}$

$x = \_\_\_$

9)
$8x = 16$

$\frac{8x}{\square} = \frac{16}{\square}$

$x = \_\_\_$

10)
$5x = 35$

$\frac{5x}{\square} = \frac{35}{\square}$

$x = \_\_\_$

11)
$9x = 54$

$\frac{9x}{\square} = \frac{54}{\square}$

$x = \_\_\_$

12)
$12x = 108$

$\frac{12x}{\square} = \frac{108}{\square}$

$x = \_\_\_$

13) $7x = 42$

14) $11n = 165$

15) $19y = 38$

16) $25q = 375$

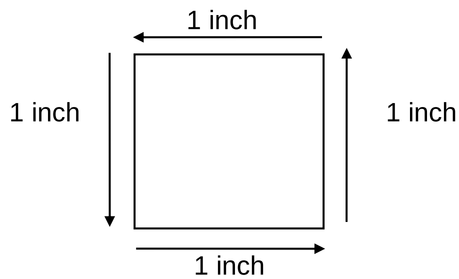
17) $80p = 800$

18) $101m = 909$

Manipulative Mathematics **Name**______________________

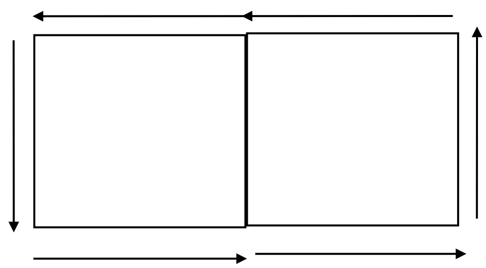
Visualizing Area and Perimeter

A color tile is a square that is 1 inch on a side. If an ant walked around the edge of the tile, it would have walked 4 inches. This distance around the tile is called the **perimeter** of the tile. The **area** of the tile is measured by determining how many square inches (or other unit) cover the tile. Since a color tile is a square that is 1 inch on each side, its area is one square inch.



Perimeter is 4 inches.
Area is 1 square inch.

1) Use 2 tiles to make a shape like the one shown below. Notice that each tile must touch the other along one complete side.



(a) What is the perimeter of this shape? Perimeter =_____

(b) What is the area? Area =____

(c) Can you make any other shape using two tiles? ____

(d) Can you find any other perimeter using two tiles? ____

(e) Record your results in the chart in #5.

2) Make all possible shapes with 3 tiles. Keep in mind that rotations and flips are really the same shape! Sketch your shapes on your grid paper, and color or shade in the squares.

(a) How many shapes did you make? ____

(b) For each shape, find its perimeter. Write the perimeter next to each shape.

(c) What is the area of each shape that you made? Write the area inside each shape.

(d) Record your results in the chart in #5.

3) Now use 4 tiles. Sketch all the possible shapes on your grid paper.
   (a) How many shapes did you make? ____
   (b) For each shape, find its perimeter. Write the perimeter next to each shape.
   (c) What is the area of each shape that you made? Write the area inside each shape.
   (d) Record your results in the chart in #5.

4) Take 5 tiles. Sketch all the possible shapes on your grid paper.
   (a) How many shapes did you make? ____
   (b) For each shape, find its perimeter. Write the perimeter next to each shape.
   (c) List all the perimeters of the shapes with 5 tiles.

   (d) Was more than one shape possible for any perimeter? ____
   (e) What is the smallest perimeter possible using 5 tiles? _____ Why?

   (f) What is the largest perimeter possible using 5 tiles? _____ Why?

   (g) What is the area of each shape that you made? Write the area inside each shape.
   (h) List all the areas of the shapes with 5 tiles.

   (i) Record your results in the chart in #5.

5) Fill in the chart below to show your results from #1-4.

Number of tiles	Perimeters Found	Areas Found
1	4 inches	1 square inch
2		
3		
4		
5		

6) Name one fact you learned about perimeter from this activity.

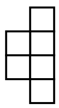
7) Name one fact you learned about area from this activity.

 Name________________________

Visualizing Area and Perimeter – Extra Practice

Find the area and perimeter of each shape.

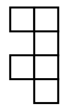
1)



area= ________

perimeter=____

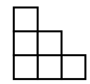
2)



area= ________

perimeter=____

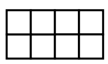
3)



area= ________

perimeter=____

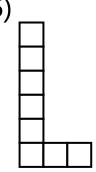
4)



area= ________

perimeter=____

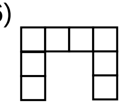
5)



area= ________

perimeter=____

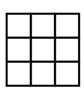
6)



area= ________

perimeter=____

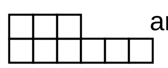
7)



area= ________

perimeter=____

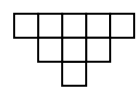
8)



area= ________

perimeter=____

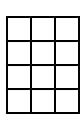
9)



area= ________

perimeter=____

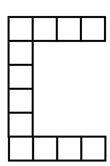
10)



area= ________

perimeter=____

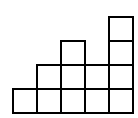
11)



area= ________

perimeter=____

12)



area= ________

perimeter=____

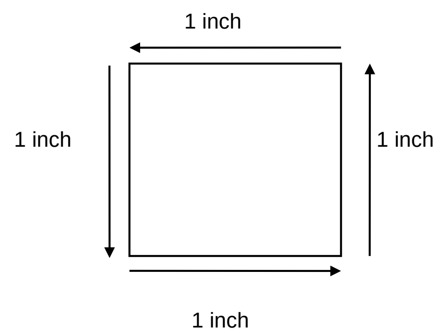
For more practice, use color tiles to make your own shapes and then find the area and perimeter. You may want to use the square blocks at the interactive website:
http://nlvm.usu.edu/en/nav/frames_asid_169_g_1_t_3.html?open=activities&from=topic_t_3.html

Manipulative Mathematics **Name**______________________

Measuring Area and Perimeter

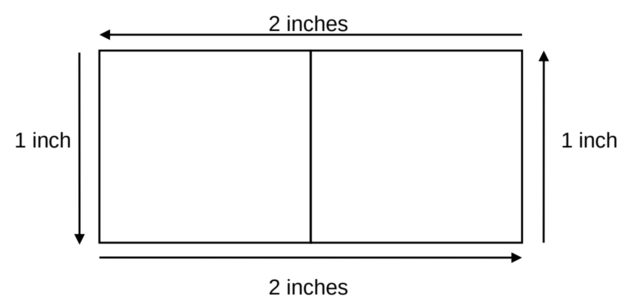
The **area** of a shape is measured by determining how many square inches (or other unit) cover the shape. The **perimeter** is the distance around the shape.

A color tile is a square that is 1 inch long on each side.
Its area is one square inch. Its perimeter is 4 inches.



Perimeter is 4 inches.
Area is 1 square inch.

If we put two tiles side by side we have a shape with area two square inches. The perimeter is 6 inches, because the distance along a side of each square is 1 inch.



Perimeter is 6 inches.
Area is 2 square inches.

1) Take your set of tiles and Shape I.

   (a) First, estimate how many tiles will be needed to completely cover Shape I. Record this in the 'Estimated Area' column on the chart below.

   (b) Next, estimate how many tiles will form the perimeter of Shape I. Record this in the 'Estimated Perimeter' column on the chart below.

   (c) Now cover Shape I completely with tiles. Count the number of tiles you used and record this in the 'Measured Area' column in the chart on the next page. Count the number of tiles along the perimeter and record this in the 'Measured Perimeter' column.

2) Repeat this process with the rest of your shapes.

Shape	Estimated Area	Estimated Perimeter	Measured Area	Measured Perimeter
I				
II				
III				
IV				
V				
VI				

3) Think about area.

(a) When might you need to use area in your everyday life?

(b) Give an example of when estimating an area is useful.

(c) Give an example of when measuring an area is necessary.

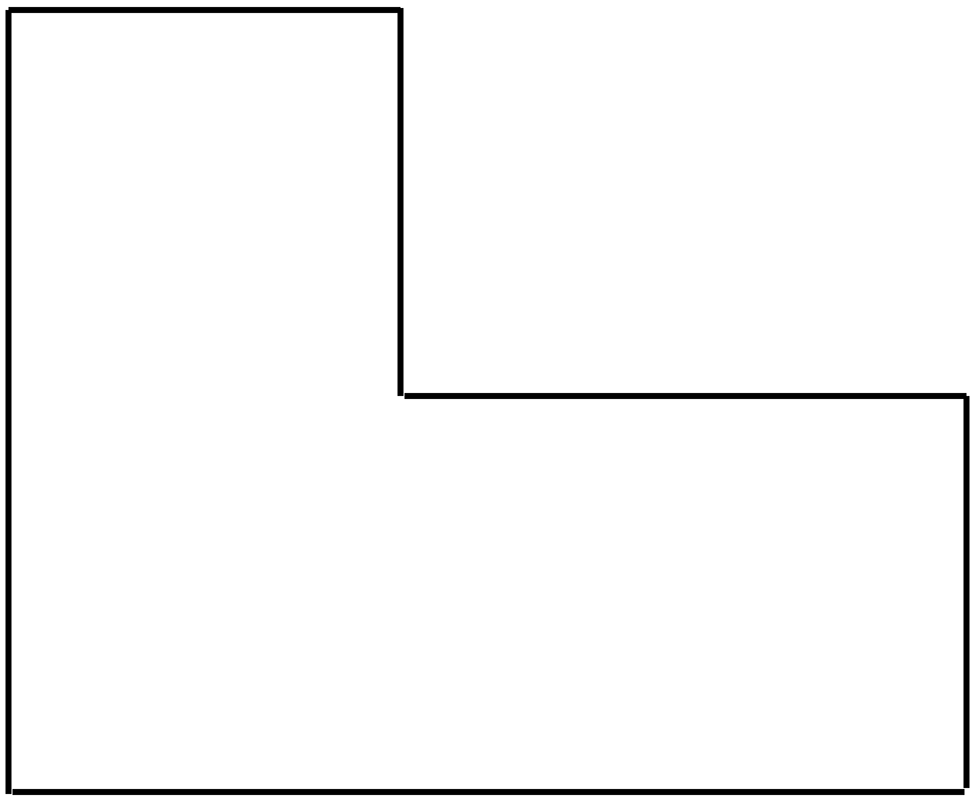
4) Think about perimeter.

(a) When might you need to use perimeter in your everyday life?

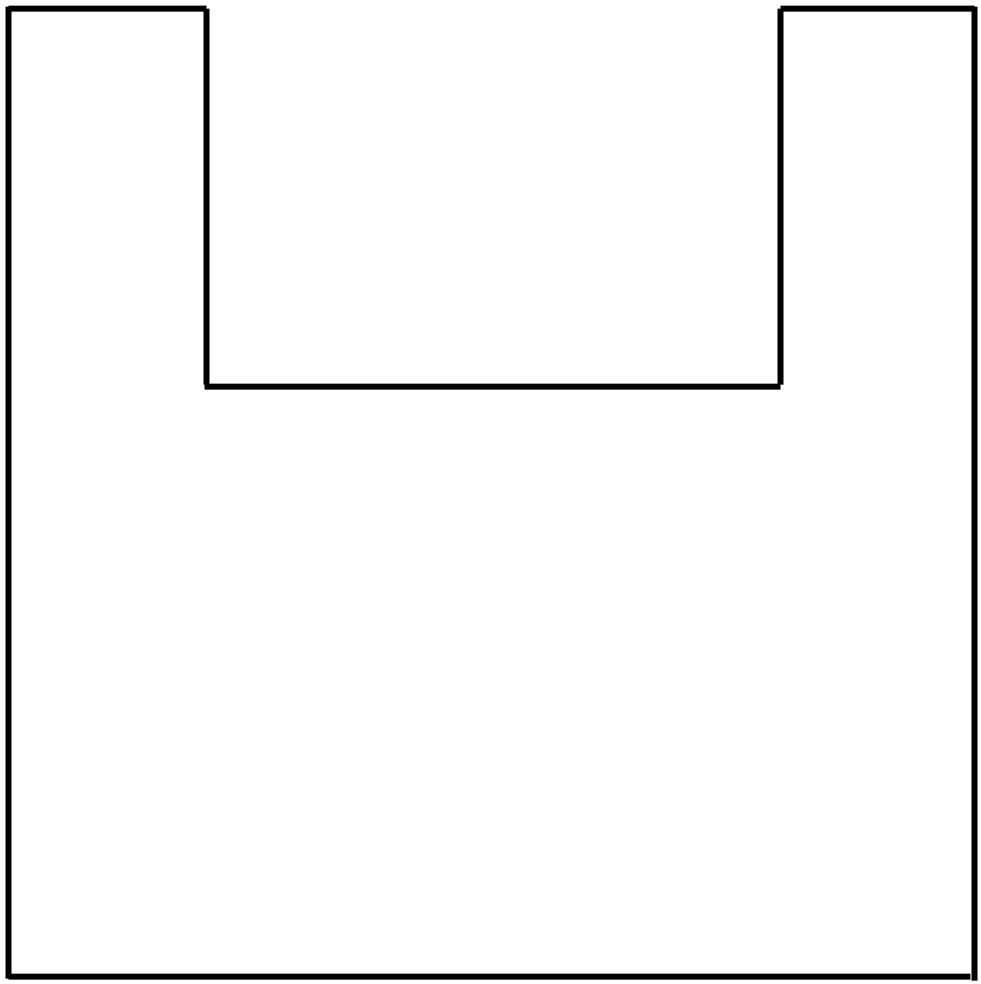
(b) Give an example of when estimating a perimeter is useful.

(c) Give an example of when measuring a perimeter is necessary.

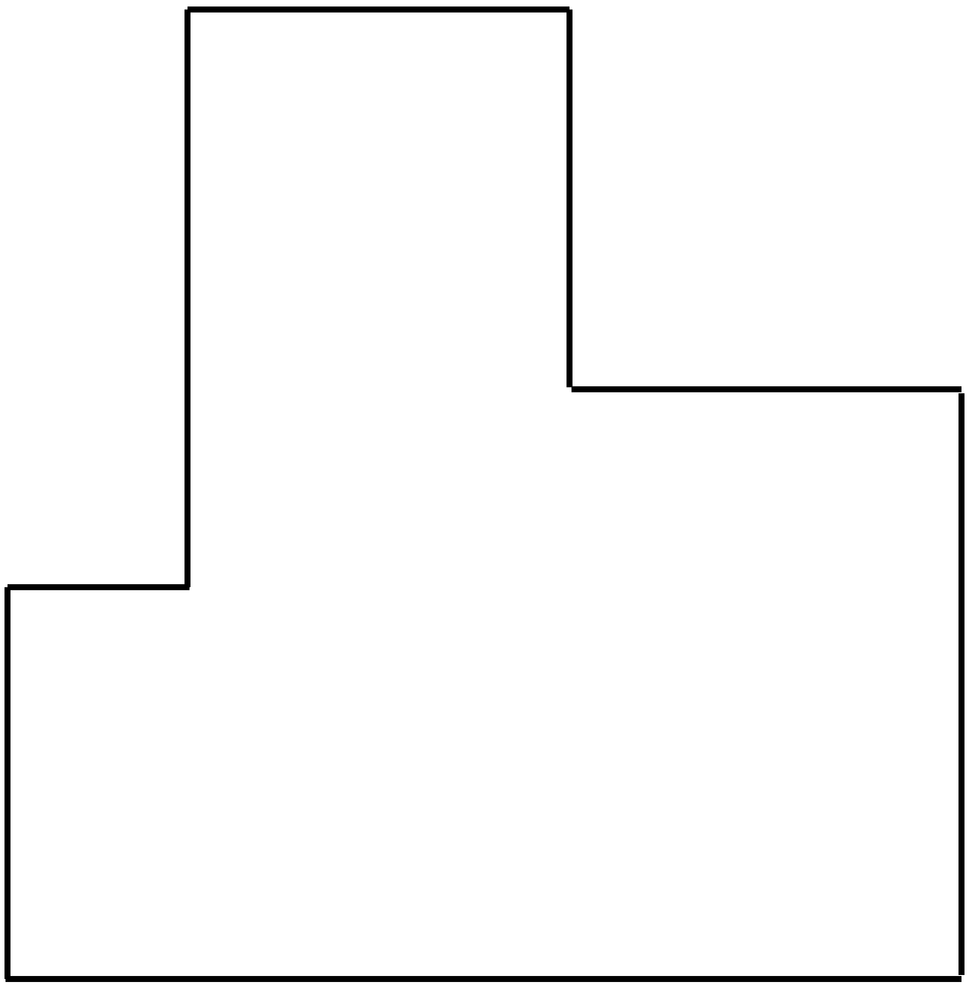
Shape I



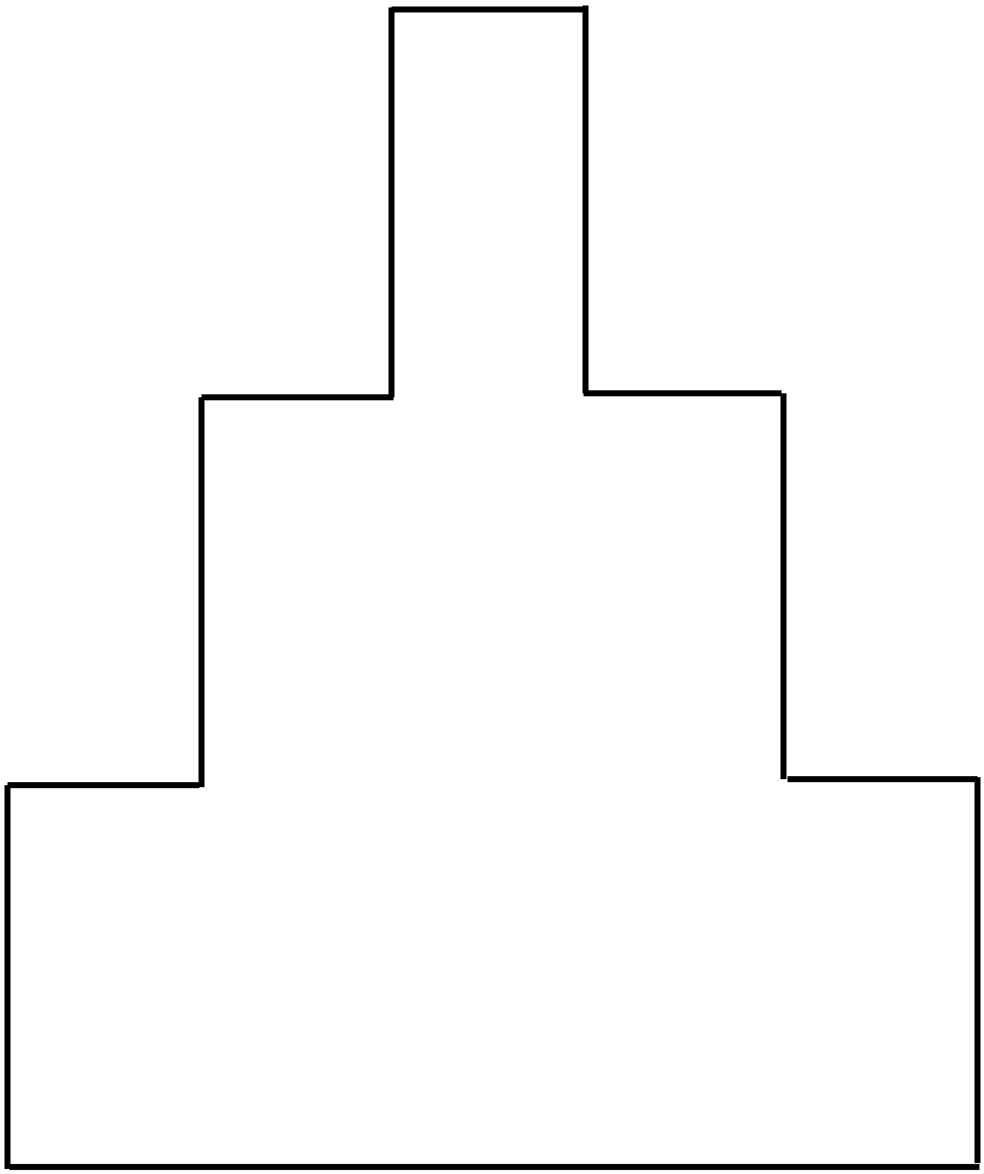
Shape ll



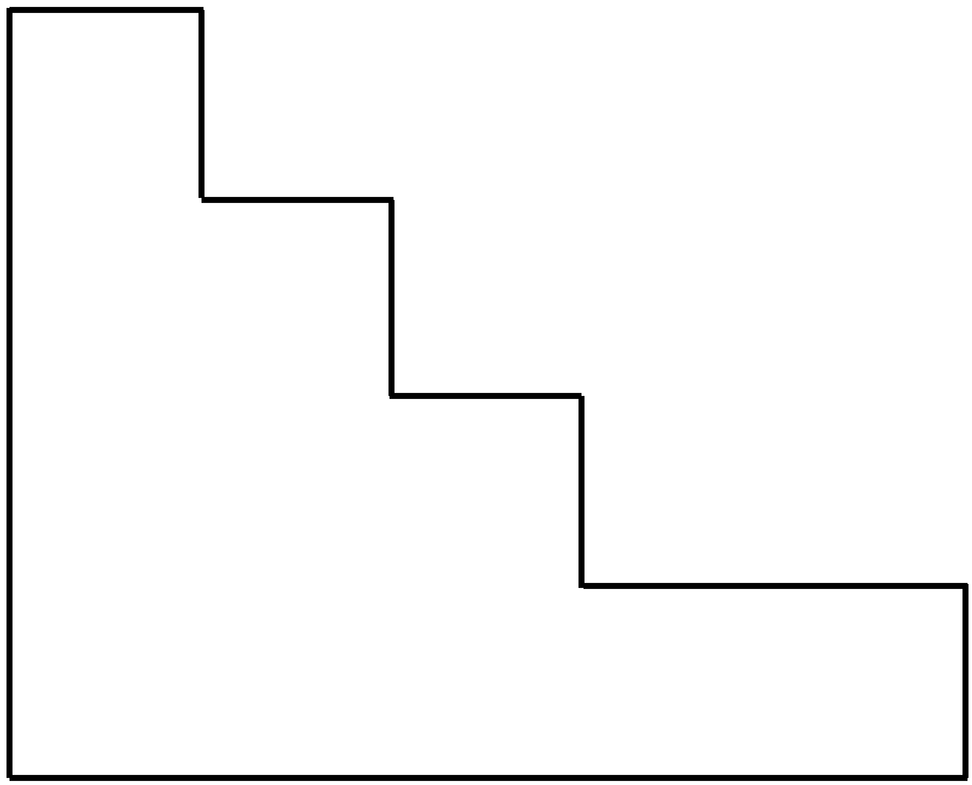
Shape lll



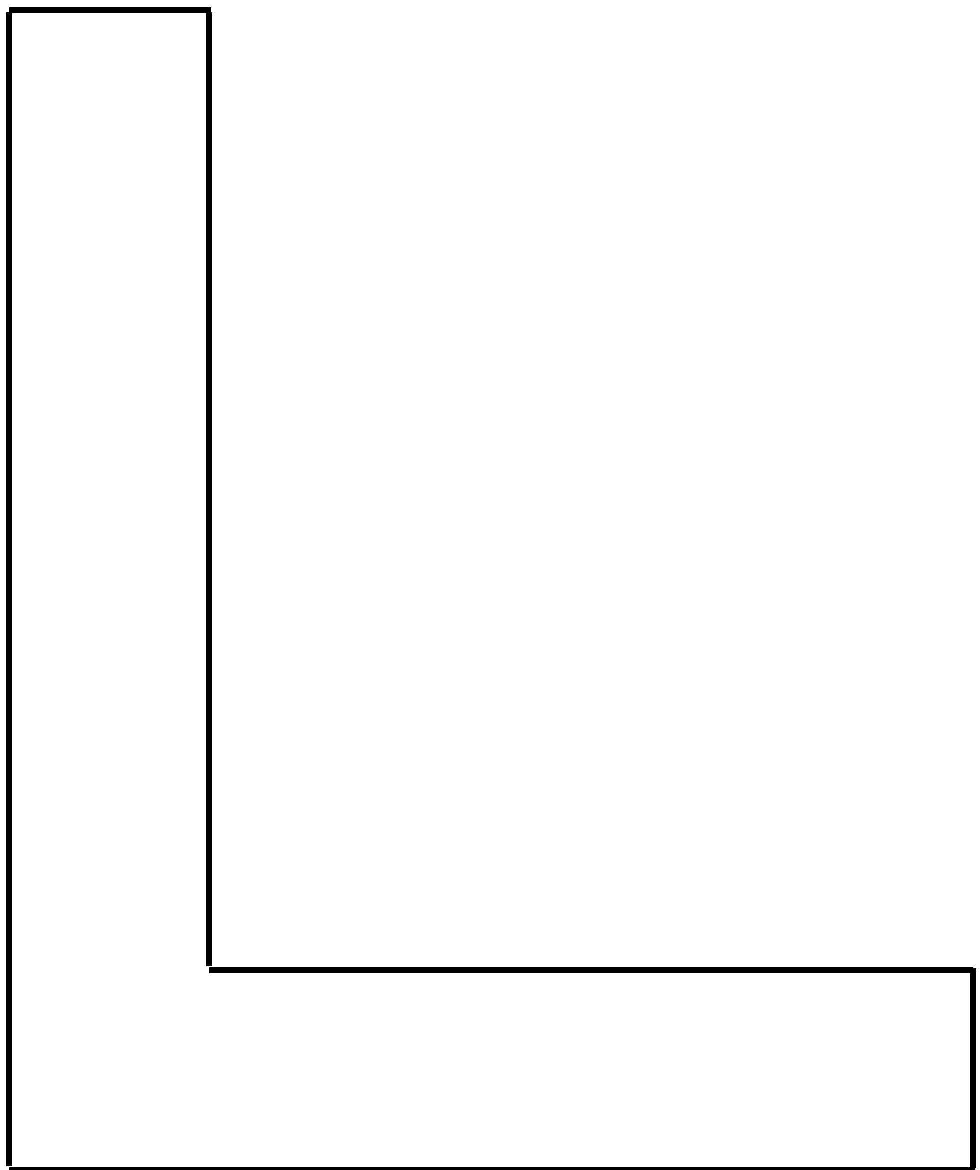
Shape IV



Shape V



Shape Vl

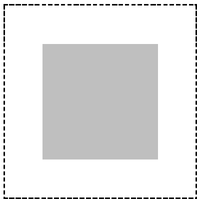


Manipulative Mathematics **Name**______________________

Measuring Area and Perimeter – Extra Practice

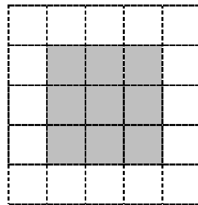
Find the area and perimeter of each shaded region, using this square [ ] as one square unit measure.

1)



Estimated area ______

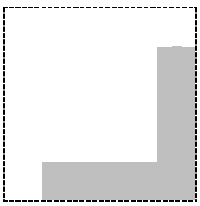
Estimated perimeter ____



Measured area ______

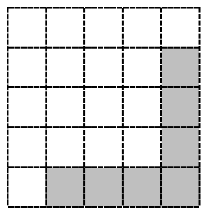
Measured perimeter ____

2)



Estimated area _____

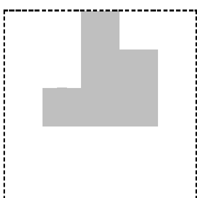
Estimated perimeter ____



Measured area _____

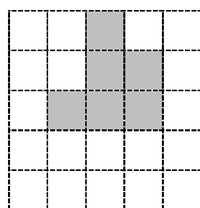
Measured perimeter ___

3)



Estimated area _____

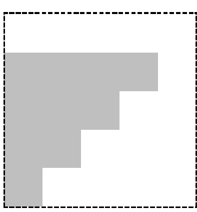
Estimated perimeter ___



Measured area _____

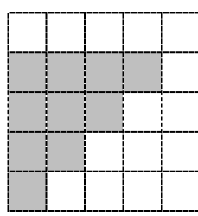
Measured perimeter ___

4)



Estimated area _____

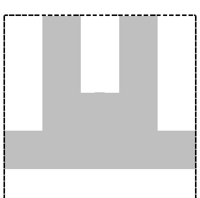
Estimated perimeter ___



Measured area _____

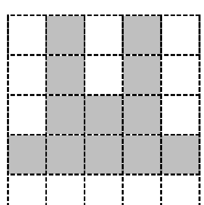
Measured perimeter ___

5)



Estimated area _____

Estimated perimeter ___



Measured area _____

Measured perimeter ___

Manipulative Mathematics **Name**______________________________

Coin Lab

Team Members:____

To start this activity, put a handful of coins into a bag. The goal is to determine the total amount of money in the bag and describe in detail the method you use.

1) How much money is in your bag?

2) Describe, in words, **the method** you used to determine how much money is in your bag. List everything you did, step-by-step, so that someone not in your group could follow your directions.

3) Show the **calculations** you used to determine the total value of the money in your bag.

Manipulative Mathematics **Name**______________________

Coin Lab – Extra Practice

Fill in the charts to calculate the total value of each set of coins.

1)

Type of coin	Number	Value ($)	Total value ($)
Quarters	3	0.25	
Nickels	4	0.05	
			$0.95

The total value of all the coins goes here.

2)

Type of coin	Number	Value ($)	Total value ($)
Dimes	5	0.10	
Pennies	1	0.01	

3)

Type of coin	Number	Value ($)	Total value ($)
Dimes	8		
Nickels	11		

4)

Type of coin	Number	Value ($)	Total value ($)
Quarters	6		
Pennies	9		

5)

Type of coin	Number	Value ($)	Total value ($)
Pennies	7		
Quarters	9		
Dimes	4		

6)

Type of coin	Number	Value ($)	Total value ($)
Nickels	15		
Pennies	3		
Dimes	8		

7)

Type of coin	Number	Value ($)	Total value ($)
Pennies	19		
Quarters	13		
Nickels	22		

8)

Type of coin	Number	Value ($)	Total value ($)
Dimes	12		
Nickels	19		
Quarters	16		

9)

Type of coin	Number	Value ($)	Total value ($)
Pennies	24		
Nickels	17		
Dimes	31		
Quarters	15		

10)

Type of coin	Number	Value ($)	Total value ($)
Pennies	29		
Nickels	14		
Dimes	23		
Quarters	35		

For more practice finding the value of a 'handful' of coins go to the website: http://illuminations.nctm.org/ActivityDetail.aspx?id=217 and choose 'Count'. If you are not completely familiar with U.S. coins, you may find it helpful to have the program display the value of each coin next to its picture.

Manipulative Mathematics **Name**_____________________

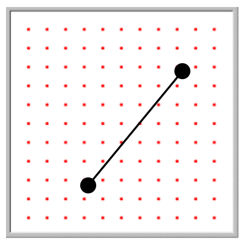
Exploring Slopes of Lines

The concept of slope has many applications in the real world. The pitch of a roof, the grade of a highway, and a ramp for a wheelchair are some places you literally see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

We will use geoboards to explore the concept of slope. Using rubber bands to represent lines and the pegs of the geoboards to represent points, we have a concrete way to model lines on a coordinate grid. By stretching a rubber band between two pegs on a geoboard, you'll discover how to find the slope of a line.

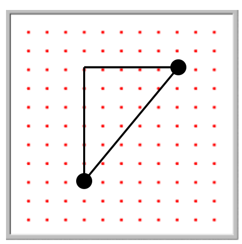
1) Let's work together to see how to use a geoboard to find the slope of a line.

(a) Take your geoboard and a rubber band. Stretch the rubber band between two pegs like this:



Doesn't it look like a line?

(b) Now stretch the rubber band straight up from the left peg and around a third peg to make the sides of a right triangle, like this:



Be sure to make a 90º angle around the third peg, so one of the two newly formed lines is vertical and the other side is horizontal. You have made a right triangle!

To find the slope of the line count the distance along the vertical and horizontal sides of the triangle. The vertical distance is called the **rise** and the horizontal distance is called the **run**.

Slope

The **slope** of a line is $m = \frac{\text{rise}}{\text{run}}$

rise measures the vertical change ↕

run measures the horizontal change ↔

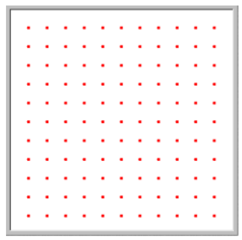
(c) On your geoboard, what is the rise? ______________

(d) What is the run? ______________

(e) What is the **slope** of the line on your geoboard? $m = \frac{\text{rise}}{\text{run}}$

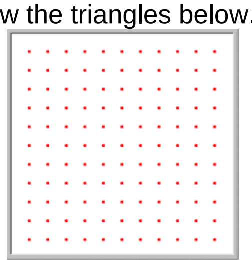
$$m = \frac{\square}{\square}$$

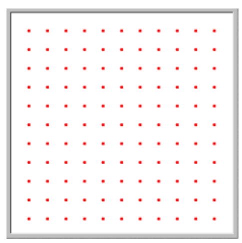
2) Make another line on your geoboard, and form its right triangle. Draw a picture of your geoboard here:

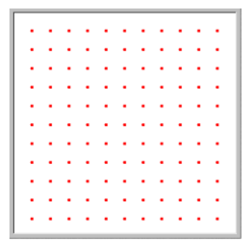


(a) What is the rise? _____ (b) What is the run? ______ (c) What is the slope? ______

3) Make 3 more lines on your geoboard, form the right triangle for each, and count their slopes. Draw the triangles below.





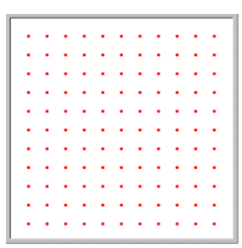


(a) Slope = __________ (b) Slope = _________ (c) Slope = __________

4) If the left endpoint of a line is higher than the right endpoint, you have to stretch the rubber band down to make the right triangle. When this happens the rise will be negative because you count down from your starting peg.

(a) Do any of your lines in exercise 3 have negative slope? ____________

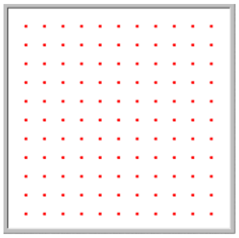
(b) Draw a line with negative slope here and calculate its slope:



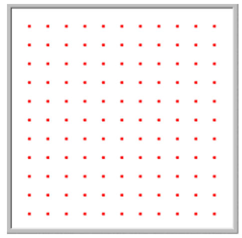
Slope = __________

5) Use a rubber band on your geoboard to make a line with each given slope and draw a picture of it.

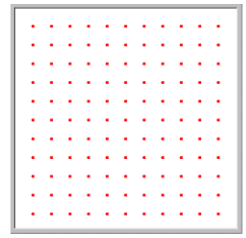
(a) Slope = $\frac{1}{3}$



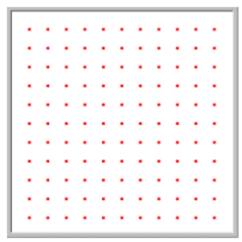
(b) Slope = $\frac{-3}{4}$



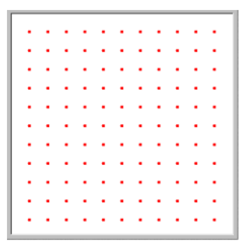
(c) Slope = 2 (hint: $2 = \frac{?}{?}$)



6) Make a horizontal line on your geoboard and draw it here. What is the slope of the horizontal line?



7) Make a vertical line on your geoboard and draw it here. What is the slope of the vertical line?



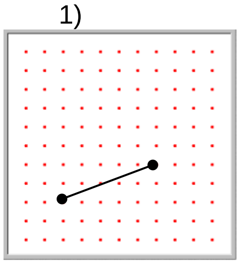
Manipulative Mathematics **Name**______________________

Exploring Slopes of Lines- Extra Practice

Sketch the rise and the run for the line modeled on each geoboard, then calculate the slope of the line.

You may want to use the virtual geoboard online at http://nlvm.usu.edu/en/nav/frames_asid_279_g_4_t_3.html?open=activities&hidepanel=true&from=topic_t_3.html.

1)

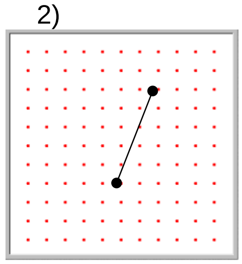


(a) rise = ____

(b) run = ____

(c) slope = ____

2)

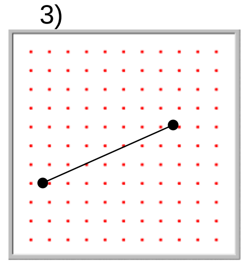


(a) rise = ____

(b) run = ____

(c) slope = ____

3)

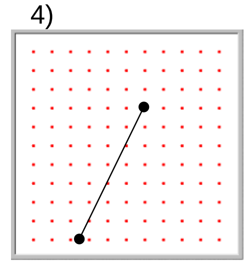


(a) rise = ____

(b) run = ____

(c) slope = ____

4)

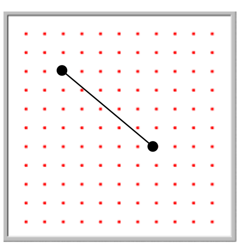


(a) rise = ____

(b) run = ____

(c) slope = ____

5)

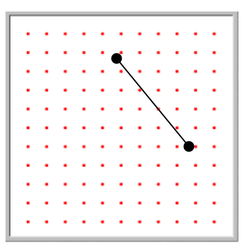


(a) rise = ____

(b) run = ____

(c) slope = ____

6)

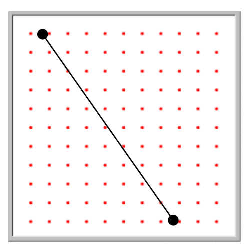


(a) rise = ____

(b) run = ____

(c) slope = ____

7)

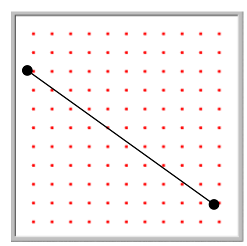


(a) rise = ____

(b) run = ____

(c) slope = ____

8)



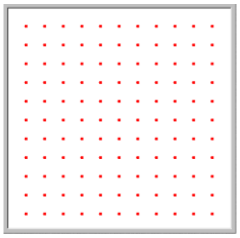
(a) rise = ____

(b) run = ____

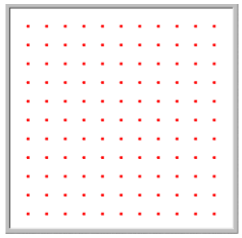
(c) slope = ____

Draw a line with the given slope.

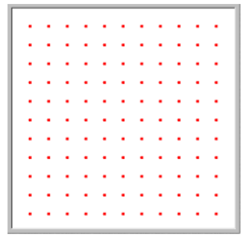
9) slope = $\frac{3}{10}$



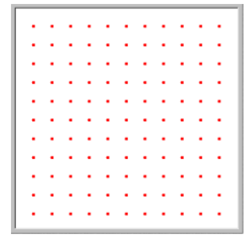
10) slope = $\frac{8}{5}$



11) slope = $\frac{-1}{6}$



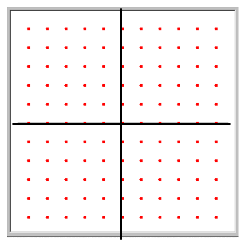
12) slope = $\frac{-7}{4}$



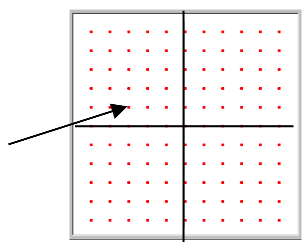
Manipulative Mathematics **Name**______________________

Slope of Line Between Two Points

1) Start with a geoboard and 2 rubber bands. Stretch one rubber band around the middle row of pegs horizontally and the other rubber band around the middle row of pegs vertically to model the x - axis and the y - axis, like this:

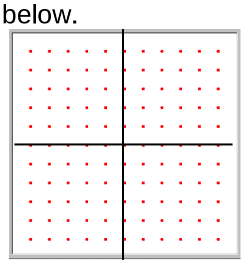


You now have a small coordinate system, with $-5 \le x \le 5$ and $-5 \le y \le 5$. Each of the pegs on the geoboard represents a point on the graph. For example, the point $-3,1$ is located at the arrow.



2) On your geoboard, make a line between the points $-3,1$ and $4,3$.

(a) Sketch it on the geoboard below.



(b) To find the rise and the run, stretch the rubber band into a right triangle, with one side vertical and the other horizontal. Draw your triangle on the geoboard above.

(c) What is the rise? ____________________

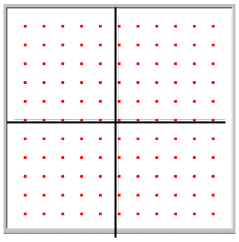
(d) What is the run? ____________________

(e) The slope is $m = \frac{rise}{run}$.

$$m = \frac{\square}{\square}$$

Find the slope of the line between each pair of points. Use your geoboard with a rubber band to model each line, then form a right triangle to find the rise and the run. Sketch each model.

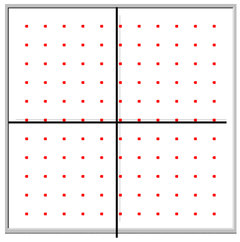
3) $-3, 0$ & $1, 5$



$\text{Slope} = \frac{\text{rise}}{\text{run}}$

$m = \frac{\square}{\square}$

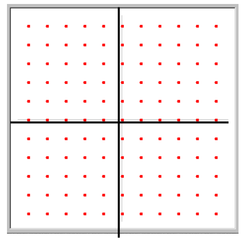
4) $-2, -4$ & $0, 3$



$\text{Slope} = \frac{\text{rise}}{\text{run}}$

$m = \frac{\square}{\square}$

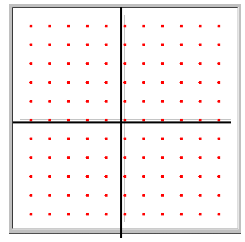
5) $-1, 2$ & $4, -1$



$\text{Slope} = \frac{\text{rise}}{\text{run}}$

$m = \frac{\square}{\square}$

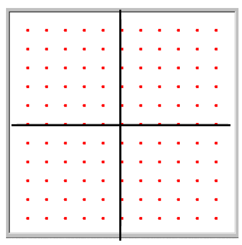
6) $-3, -2$ & $-2, -5$



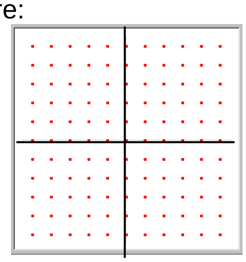
$\text{Slope} = \frac{\text{rise}}{\text{run}}$

$m = \frac{\square}{\square}$

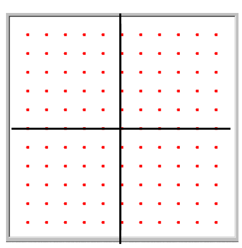
7) Start at the point $-1, -1$ and make a line with slope $\frac{3}{2}$ by counting the rise (up 3) and the run (over 2). Draw the line here:



8) Start at the point $2, 1$ and make a line with slope $\frac{-1}{3}$ by counting the rise (down 1) and the run (over 3). Draw the line here:



9) Start at the point $4, 4$ and make a line with slope $\frac{3}{4}$ by counting the rise and the run. Draw the line here:



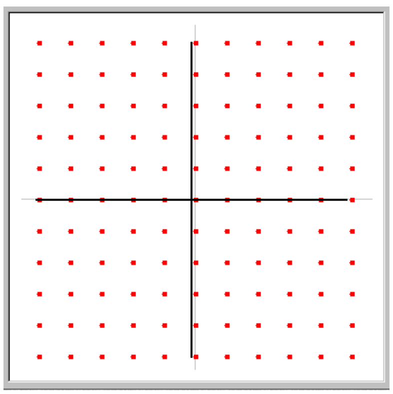
Manipulative Mathematics **Name**________________________

Slope of Lines Between Two Points – Extra Practice

Draw the line between each pair of points and then find its slope. You may wish to sketch a right triangle for each line to help you count the rise and the run.

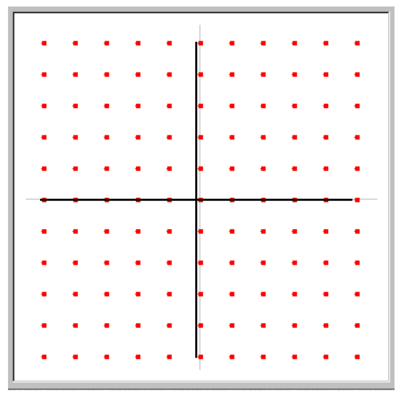
You may want to use the interactive geoboards at the website http://nlvm.usu.edu/en/nav/frames_asid_303_g_4_t_3.html?open=activities&hidepanel=true&from=topic_t_3.html.

1) $-4,0$ and $0,5$



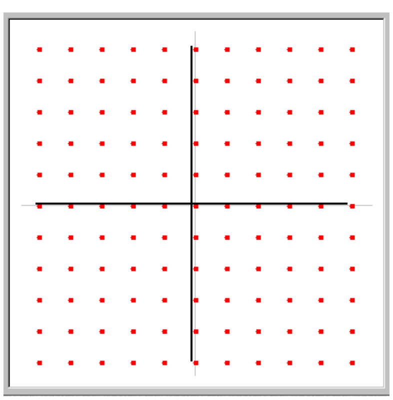
slope = ____________

2) $0,-3$ and $2,0$



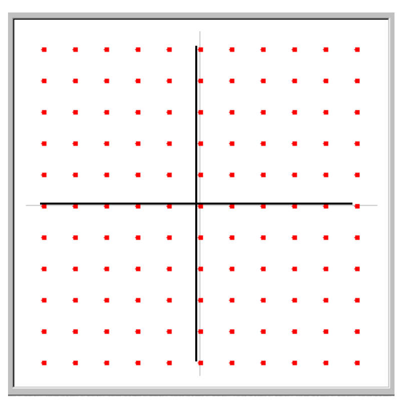
slope = ____________

3) $-2,-3$ and $1,1$



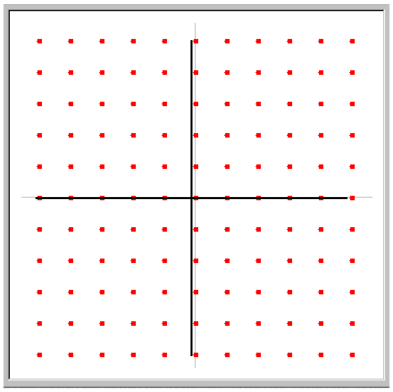
slope = ____________

4) $-5,2$ and $4,3$



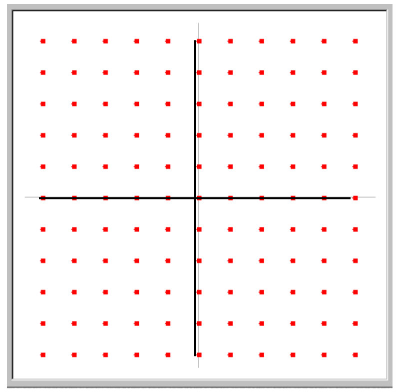
slope = ____________

5) $-1,4$ and $5,-3$



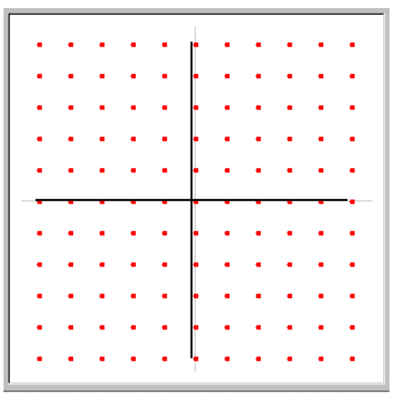
slope = ____________

6) $-4,-2$ and $4,-5$



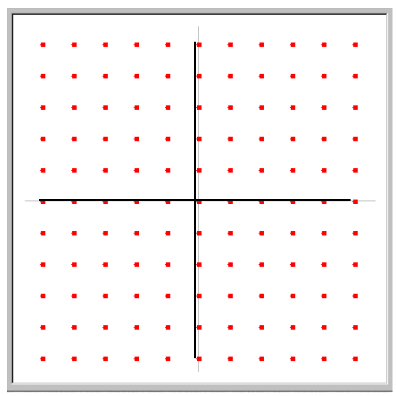
slope = ____________

7) $-3,2$ and $1,2$



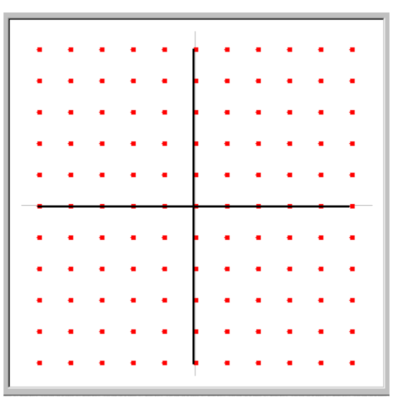
slope = ____________

8) $5,-5$ and $5,3$



slope = ____________

9) Starting at $-4,-3$ sketch a line with slope $\frac{5}{3}$.



10) Starting at $-2,5$ sketch a line with slope $\frac{-9}{7}$.