How to Succeed in Physics

(and reduce your workload)

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Veritas Tutors, LLC

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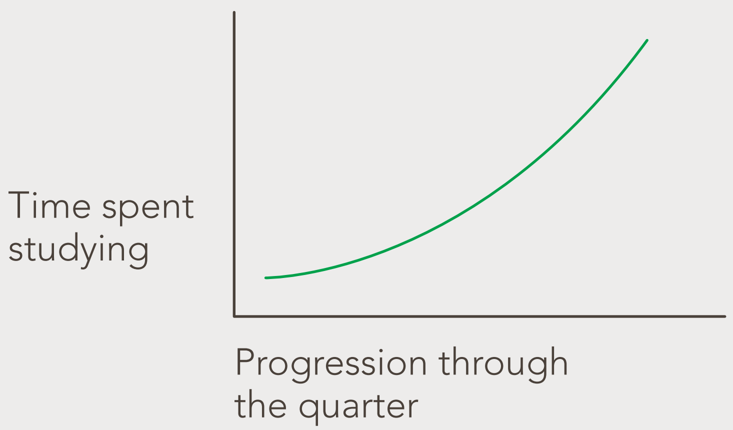
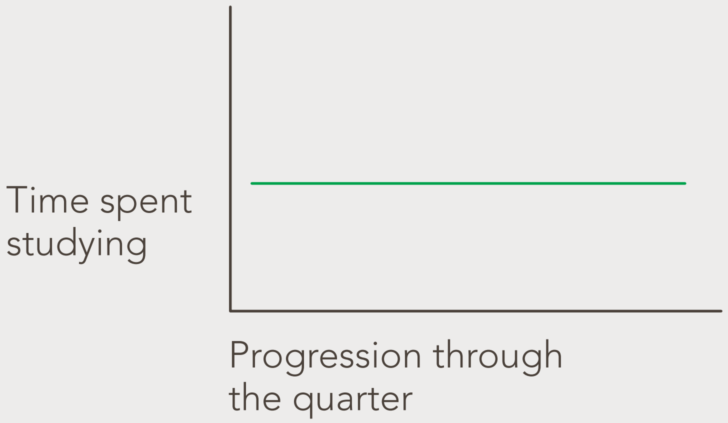
Introduction

This guide will teach you a number of concepts and techniques to help you ace your physics class with less overall work. We place a strong emphasis on understanding concepts, not just memorizing formulas and example problems. This is not only a better way to learn physics, but it is actually an immensely more efficient way to learn physics, and will make the MCAT (or SAT II or GRE, etc.) much easier as well, if that's on your horizon.

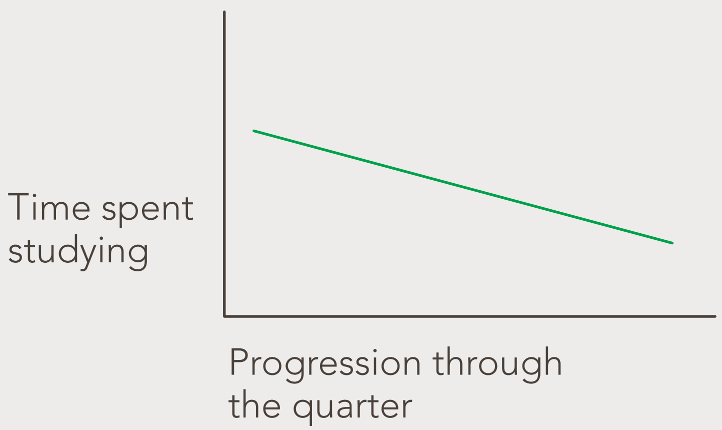
First, we need to make a note here about the relationship between study habits and physics education. As you work your way through the course, each chapter will build upon the last. As a result, if you do not keep up in the beginning, you will be in serious trouble by the middle of the class because you will have no foundation for learning the new material. By the time you get to the final exam, you will likely be completely lost. In lots of courses, students tend to take it easy at the beginning, and then cram at the end:



This will not work in physics. **You should aim to study like this:**



If you are diligent at the beginning and lay a solid foundation, you will probably actually end up studying like this:



Those of you with a solid calculus background will realize that total time studying is equal to the area under that line. You will also notice that the last graph has the smallest area, and therefore is the way to get through the course with the least work, and is also the way to assure the most success. We repeat: lay strong foundations!

For more help ensuring you have mastered all of the necessary concepts for your course, sign up for the Veritas online physics crash course at www.veritutors.com/PhysicsCourse.

Math Concepts

Math Concepts

The ideas behind the symbols

This book is going to cover math concepts that may seem very familiar to you; you might think about glossing over them. However, since physics uses math in its own special way, we believe you'll benefit from the review.

Much of math education teaches you how to manipulate symbols (for example, "x" or "Θ"). Since it's all theoretical, math professors tend to ignore what all the symbols mean; all that matters is how the symbols work with each other on the page.

Physics is the opposite. Math is only useful to a physicist if the symbols are tied to some real world concept. In class, however, most physics professors assume that everyone already understands what the symbols mean, and zoom right over the explanation.

This math review is structured to remind you how to perform some of the symbol manipulation involved in math, but is much more focused on teaching what these math equations mean in the context of real physical examples.

Connecting math to real world concepts will help you not just in physics, but also in any pursuit that requires mathematical thinking (so basically, almost any job). In any case, it is crucial to making physics easy and intuitive, and cutting your workload considerably.

Algebra

We will not go over all of the basics of algebra but rather how it will be used to solve physics problems. While these might appear very simple to you, it's important that you *know them cold*. These trusty concepts should always be in your tool belt.

The most basic algebraic concept is that anything can be done to an equation as long as it is done to both sides, and this is far and away the most used technique in physics algebra. The following boxes give a few general principles you should be familiar with.

Distributive Property	a (b+c) = ab + ac and - (b + a) = -b -a
Associative Property	a + (b + c) = (a + b) + c
Commutative Property	a + b = b + a and ab = ba

FOIL: First Outside Inside Last

(a + b) (c + d) = ac + ad + bc + bd

Fraction Math

$$\frac{(a+b)}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{x}{y} + \frac{a}{b} = \frac{xb + ay}{yb}$$

$$\frac{abcd}{ad} = bc \quad \text{BUT} \quad \frac{a+b+c+d}{ad} \neq b + c$$

$$\frac{(a/b)}{(c/d)} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

Techniques for simultaneous equations

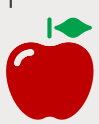
Simultaneous equations are multiple algebraic equations that share unknown variables. What does that mean? Let's look at an example:

$x - y = 2$

$5x + 4y = 19$

Both equations have an x and a y. The thing to remember here is that x and y are related to each other in both ways. Let me illustrate with a word problem:

x represents apples



y represents oranges

You have two more apples than oranges.

$x\text{-}y = 2$

Apples cost 5¢ and oranges cost 4¢. You paid 19¢ total for all your apples and orange.

$5x + 4y = 19$

How many apples and oranges do you have?

$x = ?$
$y = ?$

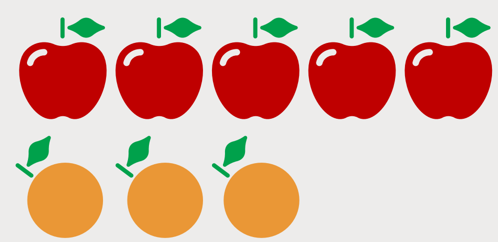
The whole point of simultaneous equations is to solve problems like this. And we can.

Principle 1: In order to solve a problem, you need as many equations as there are unknowns.

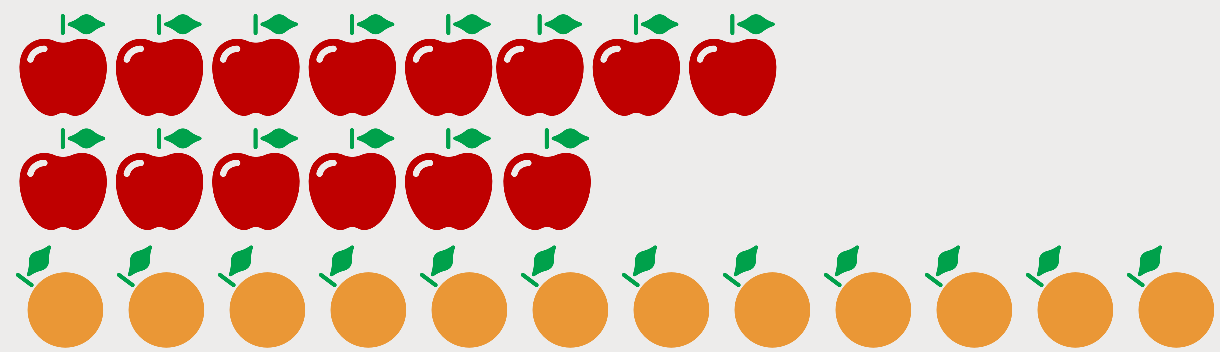
How many unknowns were in the example? Two: apples (x) and oranges (y). What if we had just one equation and lost the second?

$x\text{-}y = 2$

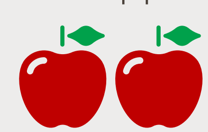
We know that we have two more apples than oranges. But that could mean 5 apples and 3 oranges;



41 apples and 39 oranges;



or 2 apples and 0 oranges!



Which one is it? We need the second equation to figure it out.

Principle 1 is true of one unknown (we only need one equation), three unknown (three equations), or 100 unknowns (100 equations!). Hopefully you won't have to solve that one.

So let's solve the example above. Since we have two equations, we will be able to solve for two unknown variables. There are two techniques to solve multiple equations, the *substitution method*, and the *elimination method.*

The substitution method

In the substitution method, we rearrange one equation and plug the result into the second equation. This is most easily seen as an example, so let's take the above equations, $5x + 4y = 19$, and $x – y = 2$:

Start with the first equation: it's simpler.

$$x - y = 2$$

Let's rearrange this for x.

$$x = 2 + y$$

OK. Now we can plug (2+y) wherever we see x in the other equation.

$$5x + 4y = 19$$

$$5(2 + y) + 4y = 19$$

Let's simplify and do some algebra.

$$10 + 5y + 4y = 19$$

$$10+9y =19$$

$$9y = 9$$

$$y =1$$

OK! We have one orange. But how do many apples do we have? Since

$$x - y = 2$$

We can replace y with 1 to get x

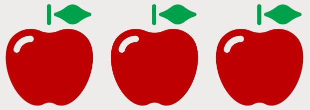
$$x - 1 = 2$$

And simply solve

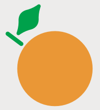
$$x = 3$$

Three apples and one orange. In other terms:

$$x = 3$$



$$y = 1$$



The elimination method

In the elimination method we essentially do simple arithmetic, but use the entire equation. The goal here is to eliminate one of the variables so that we are left with a single one-variable equation. This is also easiest to see in an example, so let's pick two new equations:

Example:

$4x + 3y = 10$

$2x + 3y = 6$:

We're going to line these up and subtract like you did in grade school

$4x + 3y = 10$

$-\ (2x + 3y = 6)$

$2x + 0\,y = 4$

We can perform arithmetic on each variable and constants:

x: $4-2 = 2$

y: $3-3 = 0$

constants: $10- 6 =4$

See how we produce a third equation with 0y? We can solve that for x.

$2x + 0\,y = 4$

$2x = 4$

$x =2$

Then we plug x=2 back into one of the original equations:

$4(2) + 3y = 10$

$8 + 3y = 10$

$y = 2/3$

We have our answer x = 2 and y = 2/3. That's the goal in the elimination method: find a way to get rid of one variable (either x or y). It's a little bit trickier, but you could also have done this with x by multiplying the second equation by 2:

$2*(2x + 3y) = 2*(6)$

$4x + 6y = 12$

Why does this help? Because now we can subtract out x:

$4x + 3y = 10$

$-\ (4x +6\,y = 12)$

$0x + -3\,y= -2$

We get a new equation

$-3\,y = -2$

or $y = \frac{2}{3}$

like above-- which is a good thing! When we plug y back into the first equation, we retrieve x:

$4x + 3\,(2/3) = 10$

$4\,x = 8$

$x=2$

It checks out!

Word problems

In your tenure as a physics student, you will be be asked to solve word problems - a lot of them. Word problems are notoriously painful, and for good reason: they ask you to think, and thinking is often painful. But if you get good at word problems, they can be fun - like a good workout.

We will start by discussing approaches that should help get you started. Here we will discuss the most basic techniques using algebraic principles.

Elementary word problem technique: solving for one unknown variable

You've got a paragraph in front of you with a lot of labels and numbers. You're pretty sure there's a vaguely worded question in there. How do you figure it out?

1. First off, *draw a picture and label everything*. This will give you a clear idea of what is going on and what you need to account for. Even if there is already a figure in the book, draw your own so you can mark it up.
2. Immediately *write all defined variables off to the side* of the page. This way you can keep track of what you know and what you don't.
3. *Figure out the missing variable* that the problem is asking you to solve for. This can be really tricky but ask yourself: what is the writer asking me for?
4. *Find an equation* that contains all of the known variables and the unknown variable. Look at a list of formulas for the chapter. For most simple word problems, you can find one that will solve the problem immediately. For more difficult problems, you'll need multiple equations (see below).
5. Once you have an equation that relates all the variables together, *solve for the missing variable.*
6. Plug in the numbers and carry out the calculation. *Keep all the units* (*kg, m/s*, etc). together too.
7. *Box your answer.* Graders love being told where to look.

Let's say you are given the following problem:

A spaceship launches with a constant acceleration. After 2 seconds the ship has gone up 6 meters.

A) What is the acceleration?

B) What is its velocity at this point?

Step 1

Write all variables off to the side

- we know t is 2 seconds

- we know Δx is 6 meters

- we also know the initial velocity v_0 is 0 meters per second because the ship started at rest

So we will put all of these in a list to the right... **aΔx = 6m**

v_0 = 0 m/s

Step 2

Figure out what variable the problem is asking us to solve for. We are actually being asked to solve for two, a, and v. Generally, we will solve for them in sequential order.

Step 3

Find an equation with all of the variables you are given and the unknown. Because you do not know the equations yet, I will list them here:

A) $v = v_0 + at$

B) $\Delta x = v_0 t + 1/2\, at^2$

C) $v^2 = v_0^2 + 2a\Delta x$

To solve for acceleration we should use equation B) because it contains our three known variables listed above (*t*, Δx, v0) and "a" so we can solve the equation with only one unknown

To solve for final velocity we should use equation A) for the same reason

So your paper will look like this:

A) $\Delta x = v_0 t + 1/2\, at^2$

t = 2s
Δx = 6m
v_0 = 0 m/s

$\Delta x = 1/2\, at^2$ — because $v_0 = 0$, $v_0 t = 0t$ and so it drops out of the equation

$a = 2\Delta x/t^2$ — Plug in your numbers and you get **a = 3 m/s** → **a = 3 m/s**

(This is now one of our variables)

B) $v = v_0 + at$ — Again, v_0 drops out

$v = at$ — Plug in your numbers and you get v = 6 m/s

Technique for multiple unknowns: Multiple unknowns require multiple equations

So you've got a problem with two unknowns in it. We solve these word problems using basically the same technique as above, except here we will need to find multiple equations.

1. Remember **our first principle**: We need *as many equations as we have unknowns.*
2. Try to limit the number of variables! Every new variable introduced by an equation will require us to add another equation. Thus if you pick an equation that contains an additional unknown, you will need another equation, and you will not have gained anything.
3. The equations that we choose will have to have at least one variable in common, or else they cannot be linked together.
4. To solve our set of equations we will use either the *substitution method* or the *elimination method*. Typically you will use the substitution method because the elimination method only tends to be helpful if both equations have the same variables.

Let's use a simple example to demonstrate:

You have twice as many jelly beans as you do M & Ms. You have 36 total pieces of candy. How many of each do you have?

1. ***Define unknown variables.*** **What is the problem asking for?**

   Number of jelly beans (let's call that J) and number of M&M's (let's call that M):

   $J = ?$

   $M = ?$

2. ***Write an equation*** **using the information in the problem.**

   We know that total candy is J + M, and that we have 36 pieces so: $J + M = 36$.

3. ***Check how many unknowns and how many equations you have.*** **If you don't have enough equations, look for a way to generate another one.**

   Notice that we have two unknowns above, J and M, but only one equation. We need another. Luckily, we know that we have twice as many jellybeans as M & Ms, so then J must be equal to 2M, or $J = 2M$

4. **Solve the equations using either substitution or elimination method**

   We will use the substitution method to solve this. Because $J = 2M$, we can just plug in 2M wherever we have a J. So $J + M = 36$ becomes $2M + M = 36$, and now we can solve because we only have one unknown in our equation:

   $2M + M = 36$

   $3M = 36$

   $M = 36/3 = 12$

We have 12 M & Ms, to find number of jellybeans just plug this back in

J = 2M

J = 2(12) = 24

We have 24 jellybeans and 12 M & Ms

5. It is generally helpful to double-check your answer using one of the original equations.

24 jellybeans + 12 M & Ms = 36 pieces of candy. ✔

Units and physics

Units give a lot of people fits. We're talking here about the labels you use, like *kg, m, s, Newtons, Joules*. Who cares, right? I just want numbers!

In fact, to a physicist, units are much more important than numbers. Why? Because units actually tell you something about the physics involved. Numbers are just digits.

Most students do not realize that they can perform algebra on units the same way they do algebra on variables. (You might have seen this in chemistry as "dimensional analysis"). This is the easiest way to check your work. Let's say you get asked to solve for distance, and you give an answer in seconds. Something went wrong!

Sounds tedious right? It can be. But it will save you a lot of time and points in the end. And you'll actually learn more physics. Trust us.

Principle 2: Units should be analyzed algebraically just as if they were variables.

Here's a great tip: Always use Standard units! By that we mean "SI" units: *kilograms* (for mass), *meters* (distance), *seconds* (time), *Newtons* (force), *Joules* (energy), and so on.

For example: If you are given a length in centimeters, convert it to meters immediately. If you do this, you never need to worry about adding inches to centimeters, or calculating mass with pounds. You can focus on solving the problem. Then at the end, give the answer in whatever units it's asking for. For example, the problem might ask for mass in grams, so solve for kilograms and then convert.

This is most easily shown through an example. So don't stress now; as with every skill, mastery comes with practice.

The standard unit for energy is the Joule, which is equal to *Kilograms* multiplied by *meters squared* per *seconds squared*

1 *Joule* = $Kg\,(m^2/s^2)$

There are many equations for energy, we will use two simple ones to demonstrate how the units will always result in Joules

---

$K = \frac{1}{2}mv2$ The equation for kinetic energy is this

$m = Kg$ The units of mass are kilograms

$v = m/s$ The units for velocity are meters per second

$v^2 = m^2/s^2$ So the units for velocity squared would be meters squared per second squared

$K = \frac{1}{2}mv^2 = Kg(m^2/s^2) = J$ Basic arithmetic shows the units of the variables work out to Joules, exactly what we want

---

$U = mgh$ An equation for potential energy is this

$m = Kg$ The units of mass are kilograms

$g = m/s^2$ The units of acceleration of gravity are meters per seconds squared

$h = m$ The units of height are meterse

$U = mgh = Kg(m/s2)m = Kg\,(m^2/s^2) = J$
Again basic arithmetic yields Joules, exactly what you want

Trigonometry

Trigonometry is the mathematics of *right triangles*, which are triangles with a 90° angle. This is important to realize because we will always want to deal with right triangles, and usually these must be constructed (often by inserting lines artificially) in order to use the tools presented here. There are only two major concepts that you will need to be familiar with, but you need to know them inside and out:

1. Trigonometric functions of *sine*, *cosine*, and *tangent*

2. The *Pythagorean theorem*

Tip: Remember that the angles inside a triangle must always add up to 180°, so if you know two, you can easily solve for the third.

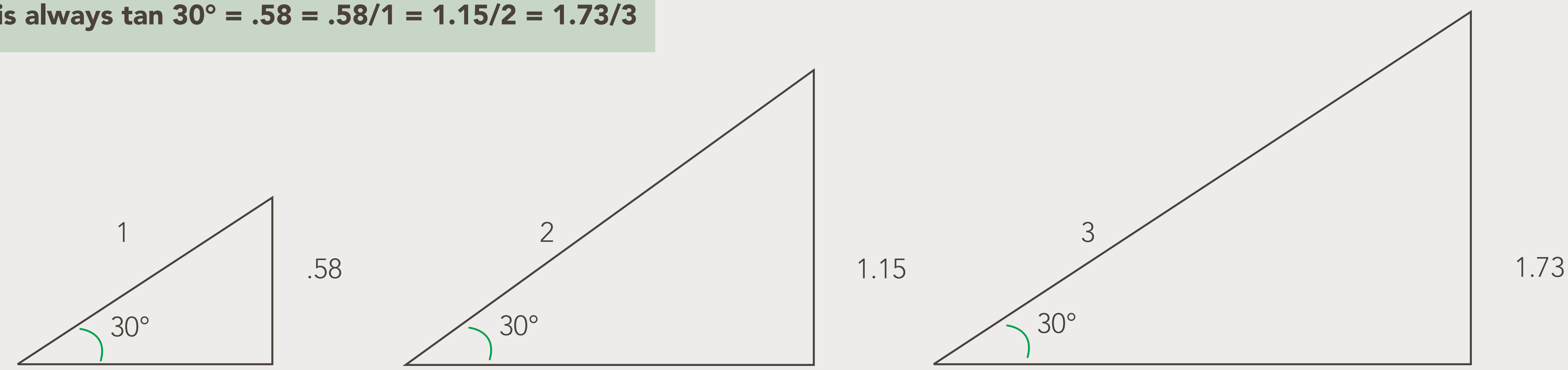
The first and most common use of trig functions: Ratios

As seen in the the three diagrams below...

Trigonometric functions can be used for many things in mathematics, but we will deal with just two: (1) *ratios* in right triangles, and (2) *wave functions.*

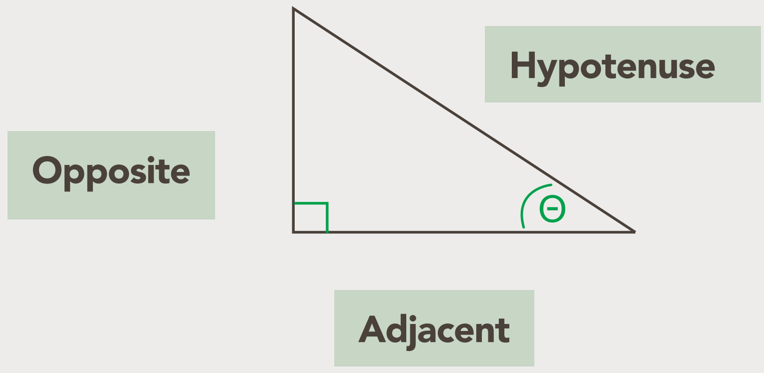
The first and most important concept that you need to understand is that trig functions are ratios of *different sides* of a right triangle. These ratio values are constant for any given angle, *no matter the size of the right triangle*. As a triangle gets bigger or smaller, each side changes in an equivalent way to the others, so that even though the *absolute values* of the sides may change, their *ratios* do not. Notice in the following diagram that as the x-component of the triangle doubles, and then triples, so does the y-component, so that the *ratio of the sides is constant.*

The ratio is always tan 30° = .58 = .58/1 = 1.15/2 = 1.73/3



S-o/h C-a/h T-o/a

"SohCahToa" is the mnemonic by which most people remember their trig functions. It tells us which trig function corresponds to what ratio of sides in the right triangle.

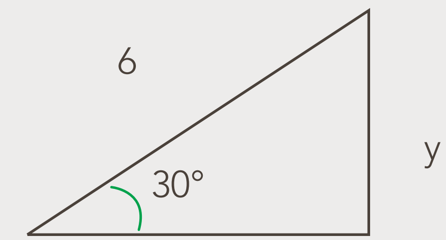


Mnemonic	Trig function	Formula	Ratio
SOH	Sine	Sin θ=O/H	Opposite over Hypotenuse
CAH	Cosine	Cos θ= A/H	Adjacent over Hypotenuse
TOA	Tangent	Tan θ=O/A	Opposite overAdjacent

You will not need to know the three other trig functions for basic physics (*sec*, *csc*, and *cot*). Whew!

Trig, meet algebra; Algebra, meet trig

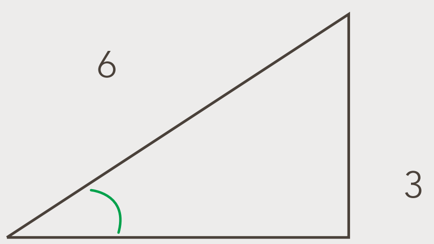
The math here is actually very simple. Each trig function for any given angle has a specific value, for example *sin* 30° is always 0.5, *sin* 45° is always .707, etc. Given this the equation is just an algebra function with three variables (x = y/z) where one of these just happens to be a trig function. Let's say we have a triangle with a known angle and one known Hypotenuse. We want to know the "opposite" side, which we'll call *y*.



Using trig functions to find unknown sides

1. Set up the equation,
   cos 30°= y/6
2. Then solve for our unknown,
   y = 6(cos30).
3. Replace the trig function with the values.
   cos 30° = .5
4. Thus
   y = 6(.5) = 3

Notice that as long as we are not solving for the angle, we are just doing algebra on our trig function. Let's say instead of being given a side and an angle we had two sides and wanted an angle.



In order to extract the angle, we need to use *inverse trigonometric functions*. These are just inverse (or opposite) of good old sine, cosine, and tangent: *arcsine* ($\sin^{-1}$), *arccosine* ($\cos^{-1}$), and *arctangent* ($\tan^{-1}$).

Using Inverse Trigonometric functions to solve triangles

1. Set up the equation:

$\sin \theta = 3/6$

2. Then solve for the angle by taking the arcsine of both sides, yielding

$\theta = \sin^{-1}(3/6)$.

3. Either using a calculator or trig table, calculate:

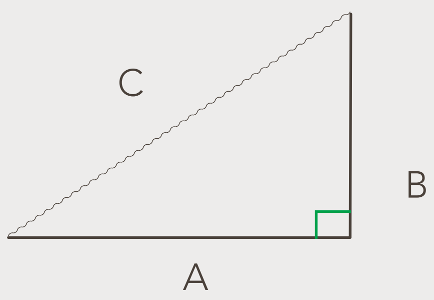
$\theta = 30°$

Conceptually, inverse trig functions are tricky. Try to think of them as questions. For example, if x=arcsin (0.5), that's like asking "What angle has a sine of 0.5?" Oh yeah-- it's 30°.

Why do we spend so much time on trigonometry? One word: vectors. More on this later.

Pythagorean theorem

The trig functions tell us what the relationship between angles and sides are for any right triangle, and the *Pythagorean theorem* tells us the *relationship between the lengths of the sides.* Given the following right triangle:



The Pythagorean theorem states that $A^2 + B^2 = C^2$, or $C = \sqrt{(A^2 + B^2)}$

This will be immensely useful when we get to vectors, because it is how you will put your resulting vectors back together to find the *magnitude* of the new vector.

The second use of trig functions: percentages, rotations, and waves

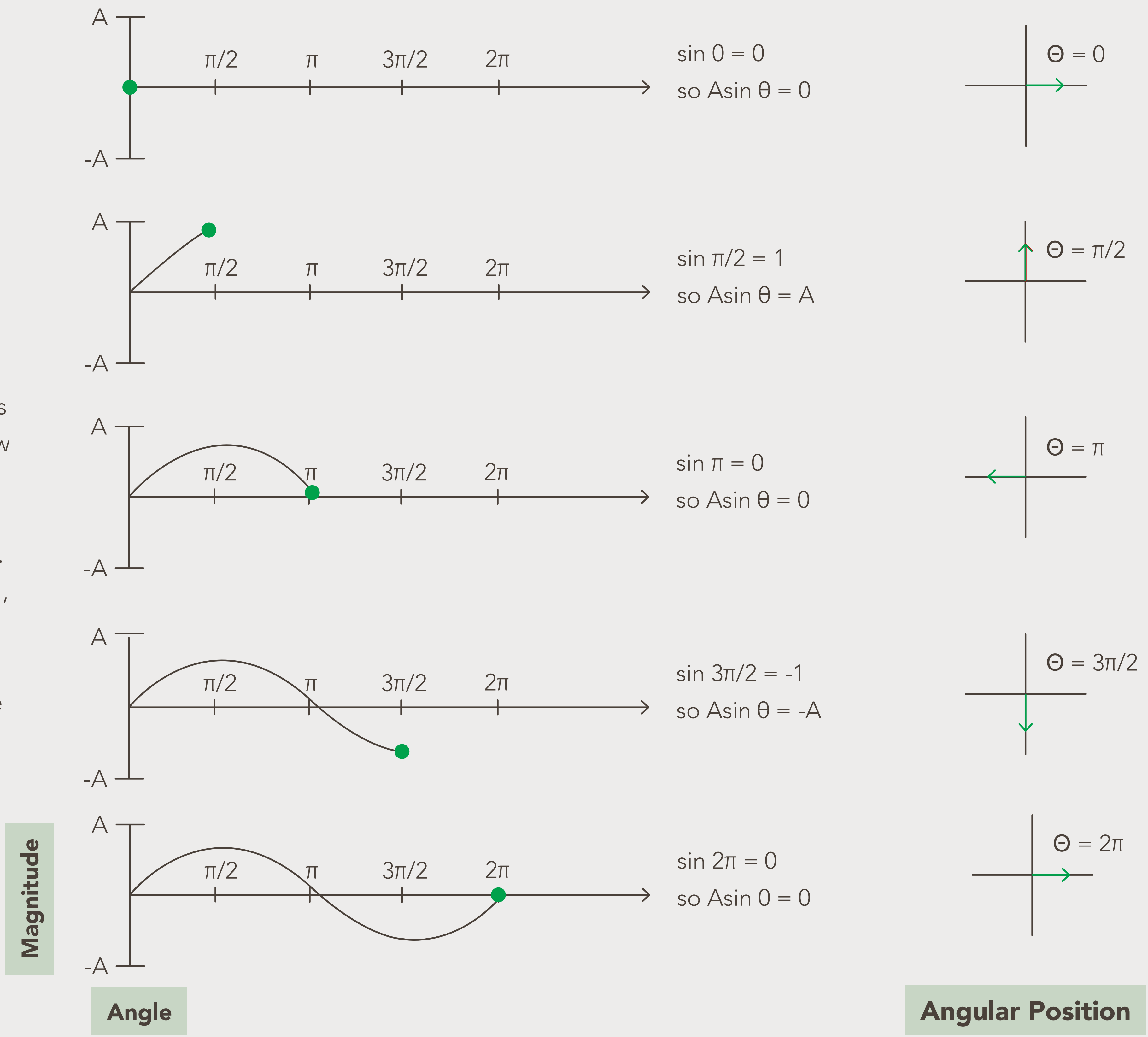
In order to understand these not so intuitive properties of the trig functions of sine and cosine (not tangent) we will first discuss the values, then the properties of a basic equation, and then a couple of pictorial explanations.

The value of a sine or cosine function can never be less than – 1, or greater than 1. That is $-1 \leq \sin \theta \leq 1$. This range has a very special property—if you multiply anything by a value between -1 and 1, this is the same as taking a proportion, and so sine and cosine can be thought of as percentage functions. That is, $A \sin \theta$ is always between the values A and -A. For example if $\theta = 30°$, then $\sin \theta = .5$ and therefore $A \sin \theta =$ .5 A, or 50% of A.

This forms the basis for a basic wave function, $x(t) = A \sin \theta$, or $x = A \cos \theta$, which just means that x, the displacement at any time t, is equal to some percentage of A, the maximum displacement. We will see how this forms a wave function below. To understand wave functions, it is best to first walk though a wave equation of one full wavelength step by step. Go through each of the steps in the following diagram, and make sure you understand how the graph relates to the equation.

Conceptualizing rotation

The easiest way to conceptualize rotation is to picture a bar with an arrow drawn on it nailed to the origin drawn to the far right. You can just think of a positive rotation of a vector by an angle θ as rotating the bar about the origin counterclockwise by the same angle. In order to conceptualize how rotation maps onto a wave function, imagine the shadow this bar would create on the y-axis as it rotates, notice where the point of the rotating bar is on the y-axis at each point. It starts at y=0, as does our wave function, then goes to y=A, as does our wave function, then back to y=0, etc.. If this shadow were graphed over time (imagine sliding a piece of paper under a pencil attached to the end of this shadow), it would generate a wave function.



Waves

Some mechanics courses discuss waves. We'll just briefly discuss a few related ideas.

Interchangeability of f, T, and ω

First some definitions, *f* (also noted as *υ*) is the *frequency*, and is a measure of how many wavelengths go by per second. T is the *period*, and is the inverse of *f*—the period is a measurement of how long it takes for one wavelength to go by. We have already seen ω, *angular frequency*. Angular frequency is just like frequency (*f*), its units are radians/second rather than wavelengths/second, requiring a 2π conversion factor to get from f to ω. The crucial message here is that these variables all represent the same thing, so *if you know one, than you know all three*. Don't forget this because often the problem will give you one when you need another such as giving you T, when you need *υ* for your equation.

The wave equation

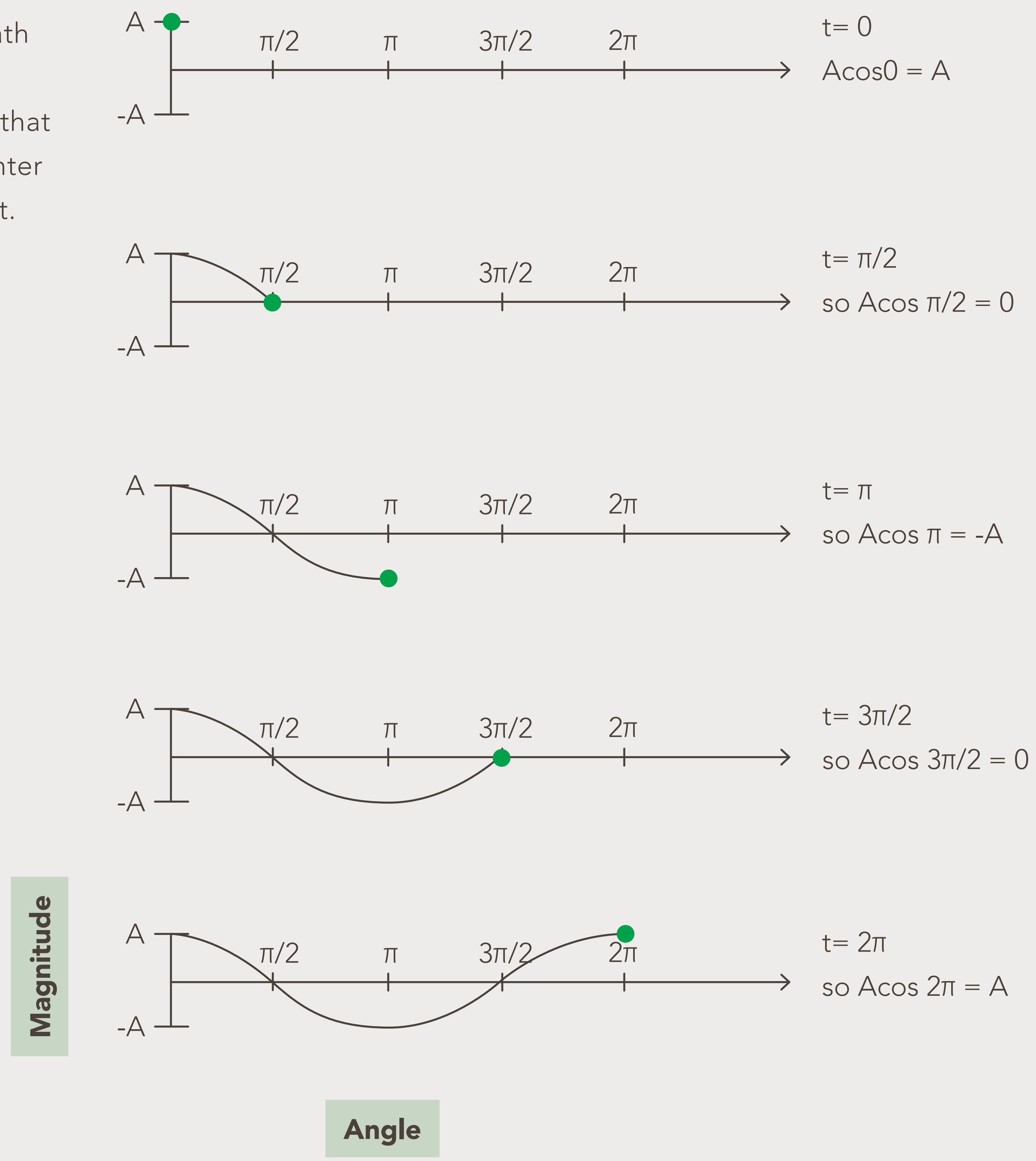
The standard equation that you will learn to deal with waves looks very intimidating, but is actually quite simple. We are going to deal with this by dissecting the parts of the equation, and then showing how a simple version of the equation relates to the graphical representation. Last we will add in all the pieces so you understand the whole thing. The wave equation will look like this $x(t)=A\cos(\omega t + \varphi)$.

This is no different than the math of waves introduced above, however. Remember that $\omega t = \theta$, just as $vt = x$. φ is just an adjustment angle, and can be ignored for our initial assessment (it is usually 0 anyways). A is the maximum displacement, and x(t) is the displacement at any time t.

To start simple, let's say that $\omega = 1$ rad/s so that $\omega t=t$, and let $\varphi=0$. This simplifies the equation down to just $x(t)=A\cos t$, and this is shown below.

variable	Conversion to one	Conversion to other
f or *υ*	1/T	ω/2π
T	1/*f*	2π/ω
ω	2π*f*	2πT

As can be seen in the graph this is just as the one we did in the math review section, but with a cosine rather than a sine. Granted the equation can be more complicated than this, but not by much. All that ω will do if it is not 1 rad/s, is stretch out the graph or squish it tighter together. All that φ will do if it is not 0 is shift the graph left or right. This can be used to model all kinds of motion, like a spring, whose maximum displacement is A from its equilibrium point.

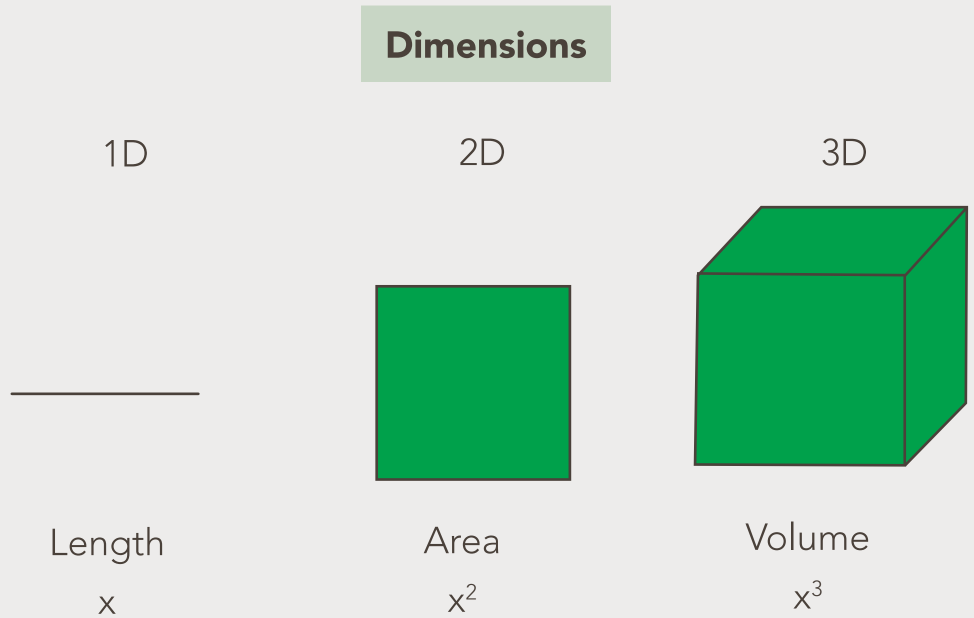


Geometry

There are two important concepts from geometry that you will need to know for physics: *dimensions*, and the *meaning of π*.

Dimensions

A dimension is a degree of freedom, and this is important in two aspects, both in understanding the relationships of length, area, and volume, and in understanding how to dissect vectors. A *degree of freedom* is a way in which *something is free to vary*, so for example, we live in three dimensions, which just means that we can move forward-backward, up-down, or left-right. In other words we can vary our position in space in three ways.



An easy way to figure out the dimensionality of a given space is to look at the power of the variable. Let's say we have a length we'll call X. This is one-dimensional; its exponent is 1. If we then made a square with two of these lines, it would be two-dimensional; its area would be X^2 - notice its exponent is 2. If we were to make a cube with three of these, then the formula would be X^3, now it is three-dimensional and the exponent is 3.

Obvious, right? The thing to realize is that **dimensionality is the key to physics.** Remember when we said that units are more important than numbers? Units are a form of dimension. We're used to thinking of dimensions as length, area, and volume, but the same idea applies to everything: seconds, kilograms, etc. The next time you look at a formula, keep its dimensionality in mind; you should be able to understand it better and remember it more easily.

The other reason dimensions are important is that they lay bare the relationship between vectors and algebra. Essentially, algebra only works in one dimension. However, vectors can represent quantities of any number of dimensions, usually two, sometimes three. This means that vectors need to be broken into one-dimensional pieces, in order to use simple algebra. Furthermore, for most of mechanics dimensions do not interact with each other. Weird, right? But true. This is why we will break vectors into components using trig functions.

Demystifying π (Pi)

The number π (Pi) is often exalted as the most significant number in the universe. Why? Probably because when you look at nature, *it shows up a lot*. That's because it is the *ratio of the circumference of a circle to the length of its diameter.*

Let's say I take a wheel of diameter 1 meter. (It's a monster truck wheel). I roll it once along the ground. Where does it end up after one rotation? 3.14 meters away. That's Pi.

So what? It's not that some great mathematician sat down and *made up* the number. He just went out and measured it, just like you could do no matter where you are, with any circle. The relationship will always be the same: 3.14. It's like a calling card of our universe. Pretty cool, right?

our tire at the beginning | **our tire after one rotation** | **our tire after 2 rotations**

Our tire has gone the distance of its circumference in one full rotation, twice the circumference in two and so on

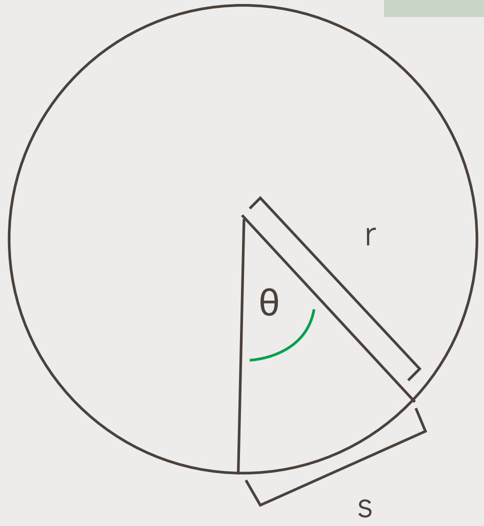
$$C = \pi \qquad \text{Distance} = 2\pi\, d$$

Because d = 1, then c = π which is 3.14

Demystifying radians

Radians are like Pi's little brother. Let's say you want to measure the length of an arc (around the rim of a circle). For example, you cut yourself a piece of apple pie. What's the relationship between the angle of the piece, and the length of its crust?

Arc length = radius* angle $s=r\theta$



The length of the arc divided by the radius is the number of radians. For example: the radius of the pie is 10 centimeters. I cut out a piece of pie, and measure its crust. It carves out an arc of length 8 centimeters. What's the angle of piece? It's just

$s/r = 0.8$ radians.

That number might not mean much to you. But we can convert radians to degrees. There are 360 degrees in a circle, right? How many radians do you think there are? 2 Pi or 6. 28 radians.

Therefore

0.8 radians * 360/6.28 = 45.8 degrees

45 degrees is half of 90 degrees. Easier to visualize, right? Have another slice!

Calculus *(optional section)*

No need to fear

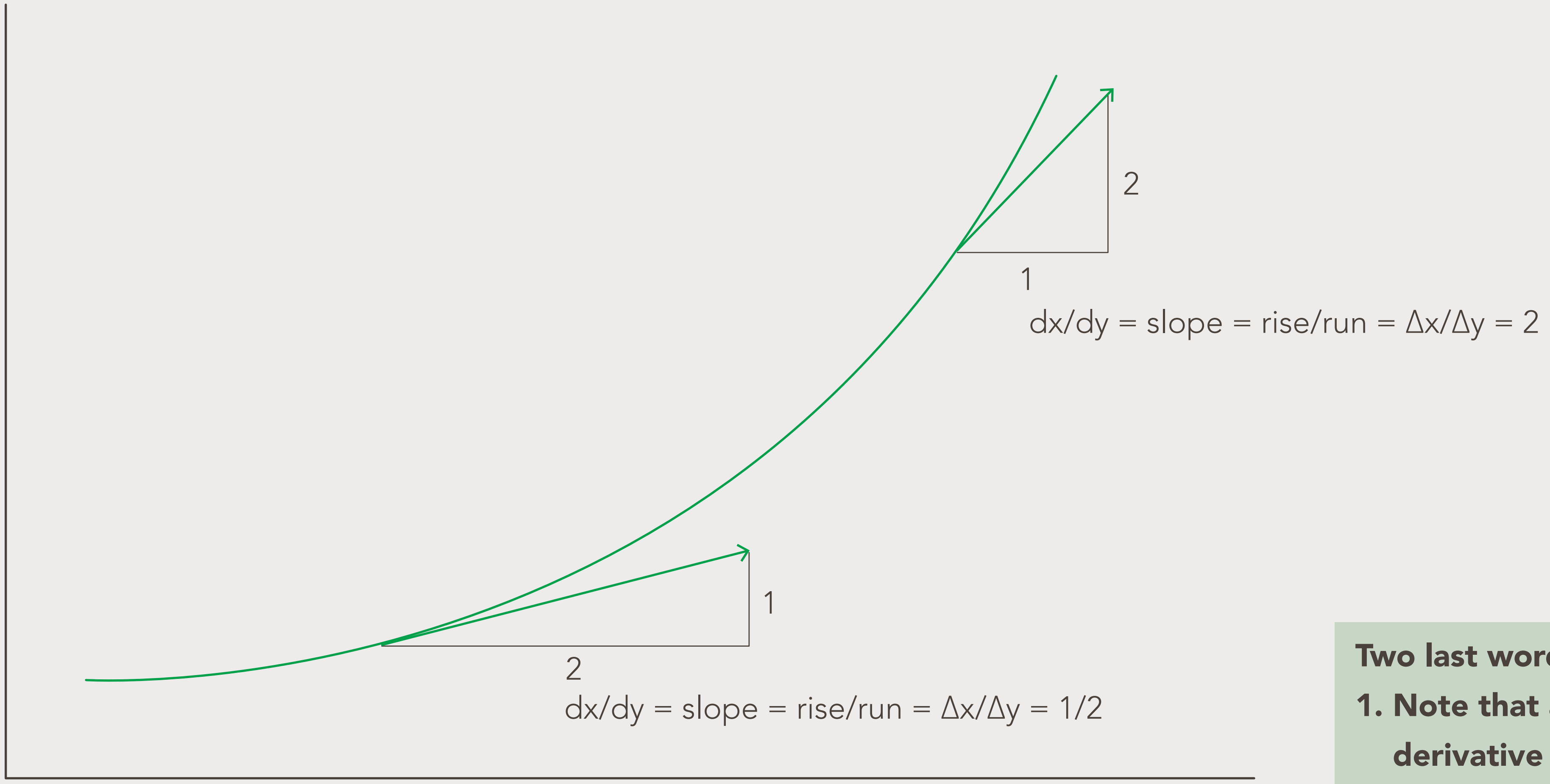
Many students are very afraid of calculus, even though it is actually not that complicated. They think that calculus is going to be a big part of physics and that they are going to have to do complicated derivatives and integrals. Calculus actually plays a very small role in most of university physics, and there are only a couple of very simple ideas that you should be familiar with. Often a professor will use calculus to derive a formula in lecture, but you probably don't need to understand the derivation to succeed in the class. Instead you just need to be able to use the formula at the end to solve word problems. What you do need to know are: (1) *the basic operations for performing simple derivatives and integrals*, (2) *what derivatives and integrals mean, and* (3) *how they are represented in a graph.*

A derivative is a rate of change

Derivatives signify a rate of change, and are analogous to rates of change in algebra, just more precise. This means that whenever you have a rate in physics (velocity for example) it can be represented as a derivative. Velocity is equal to the change in distance divided by the change in time; as a formula, we can say $v = \Delta x / \Delta t$. This is analogous to the derivative; we can also say $v = dx/dt$. The advantage of using a derivative is that this rate of change is instantaneous. The two main derivatives that you will be responsible for in physics are velocity and acceleration ($a = dv/dt$). Velocity is a change in distance over time, and acceleration is a change in velocity over time.

A derivative is the slope on a graph

A derivative is the slope of a line tangent to a point on a graph. A tangent is an arrow coming off a point on a graph that goes where that single point would go if it went straight. The arrows below are tangents:



Two last words on derivatives:

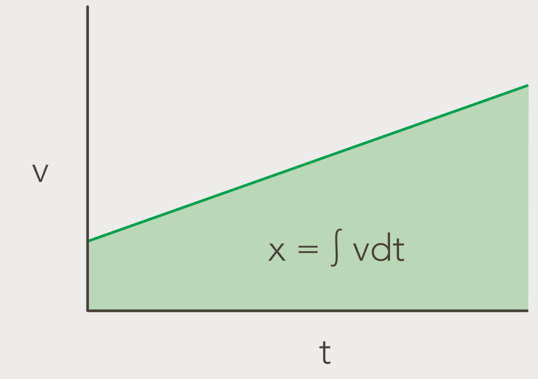
1. **Note that at any maximum or minimum, the derivative will be 0.**
2. **Do not forget that the derivative of a sine wave becomes a cosine wave and vice versa.**

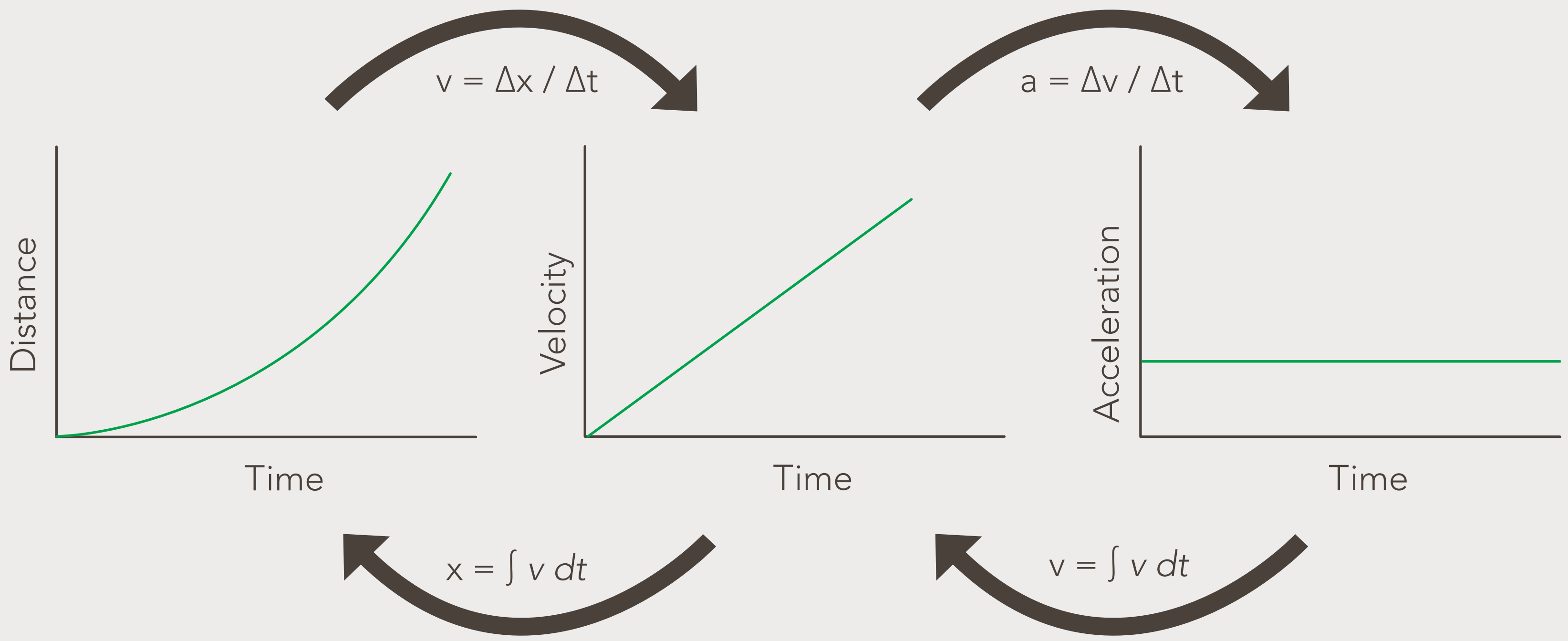
An integral is an antiderivative

This is the fundamental theorem of calculus: *an integral is an antiderivative.* This means that an integral is nothing but the summation of the change that has taken place. The non-calculus equivalent is just a discrete sum (represented by Σ). This is also the inverse operation of a derivative, so if velocity (v) is the derivative of position (x), then position would be the integral of velocity. $v = dx/dt = \Delta x/\Delta t$, and so $x = \int v dt = \Sigma\, v\Delta t$.

An integral is the area under a curve

Graphically, an integral is represented by the area under a line or curve on a graph. Because $x = \int v dt$, this means that total displacement would be the area under the graph of velocity with respect to time. Displacement (distance traveled) is represented in the green area of the graph below.





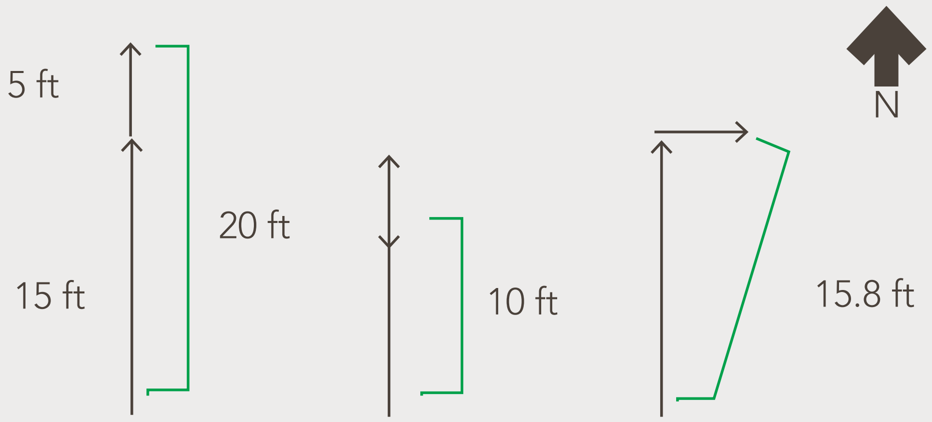
Vectors and scalars

Vectors are probably the first new mathematical concept that you will encounter in physics, and are usually not very well explained. You will be told, "vectors are quantities with both magnitude and direction," and, "scalars are quantities with just magnitude." These statements are rarely unpacked, however, and that's bad because vectors are perhaps the *single most important mathematical tool in general physics.* Scalars are represented as regular variables, whereas vectors are represented with little arrows above them (such as force and acceleration in the force equation below). We know that F and a are vectors because they have arrows; m is a scalar because it does not.

$$\vec{F} = m\vec{a}$$

So, let's unpack the concept, what exactly does it mean to have both magnitude and direction? What does it mean to just have magnitude? To say that something has both magnitude and direction means that we can't just do arithmetic on the quantity, because we have to take direction into account. Something that has magnitude but no direction is just like any number that you are accustomed to dealing with in arithmetic or algebra. This is most easily understood using examples.

A great example of a vector is walking distance. Let's say I am standing right next to you, and I walk 15 feet. I stop and then walk 5 feet, how far away am I from you now? The answer to this question depends on the *directions* that I walked, not just the *magnitudes* of how far I walked.

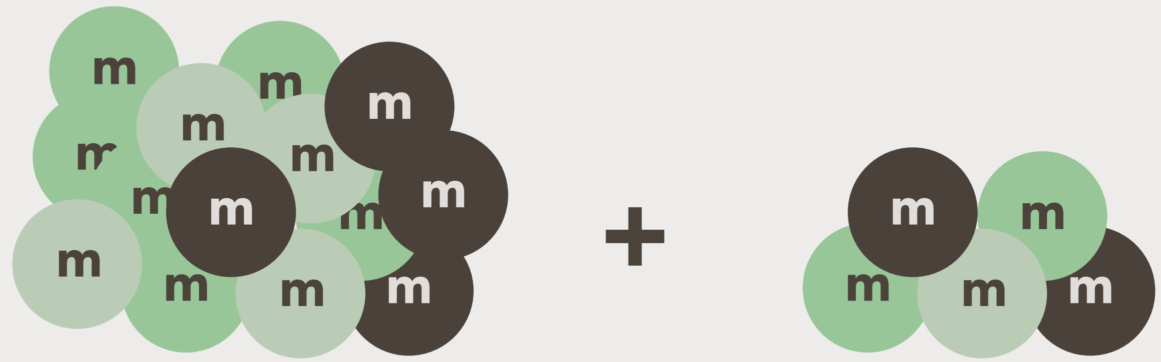


Addition of vector depends on direction

If I walked 15 feet north, and then 5 feet north, I would be 20 feet away. If I walked 15 feet north, and then 5 feet south, I would be 10 feet away from you. However if I walked 15 feet north and then 5 feet east I would be 15.81 feet away (given by the Pythagorean theorem).

A vector can be said to have *magnitude* and *direction*, because it can vary over a space with more than one dimension, so the direction in which it is changing must be specified. This means vectors are *path-dependent*—the final outcome depends upon not just the size of the steps, but also upon the specific path they traversed.

On the other hand a scalar only has magnitude because it can only vary in one way (in one dimension). That is, a scalar can only increase or decrease. A great example of a scalar is any sort of quantity. Let's say I give you 15 M&M's, and then I give you 5 more M&M's. How many M&M's do you have? The answer to this question is very straightforward because you do not need to take direction into account like above, so you have 20 M&M's, it would be impossible to have 15.81 M&M's because the Pythagorean theorem is not relevant for a one-dimensional quantity.

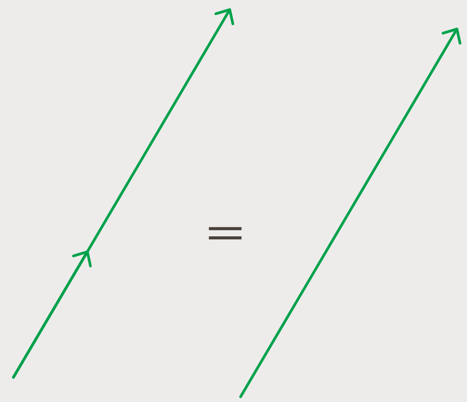


Addition of scalars depends only on magnitude

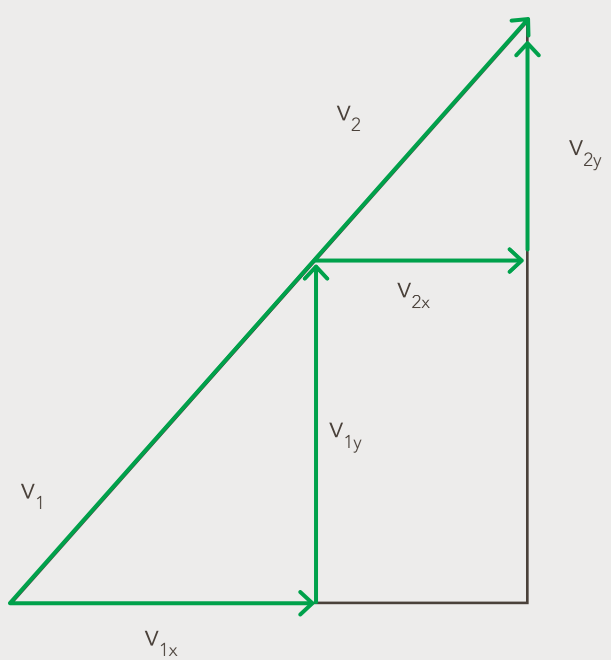
Understanding this distinction is important because algebra can only be done on one-dimensional quantities. Most of the math that you are probably familiar with is done with scalars. Since vectors are multi-dimensional, they can be tricky if you aren't careful. The reason is that they cannot just be automatically plugged into the math equations that you will learn, but rather they must first be broken into components. I will illustrate this in a couple of ways, because it is crucially important to understand this concept to succeed in physics. Once vectors are broken into two or three one-dimensional components, these components do not interact and can be treated as completely separate entities (with a couple exceptions like cross products).

Visualizing vectors

Pictorially, vectors add "tip to tail." Like this:



You will not usually be adding vectors pictorially like the above, but mathematically. To do so, you will need to break vectors into x- and y-components. The diagram below shows how this works:



First break vector 1 (v1) into its two components ($v1_x$, $v1_y$), and do the same for vector 2 ($v2_x$, $v2_y$). You can then add these components ($v1_x + v2_x$, and $v1_y + v2_y$) to yield the same final vector. It is the equivalent triangle whether we add the legs or the Hypotenuses. To get the magnitude of the resulting vector, we would of course have to use the Pythagorean theorem (demonstrated below).

Vectors and trigonometry

The pictures should show you the relationship of vectors and right triangles. Mastering the trig functions is especially important because they are used to cleave vectors into one-dimensional components so they can be dealt with using algebra and arithmetic. Once vectors have been broken into their respective components, a problem proceeds as though it were two separate and independent problems—one problem on the x-axis, and one problem on the y-axis. Then at the end of the problem, these results are put back together using the Pythagorean theorem to give the magnitude of the final vector. A *vector will always be the Hypotenuse* of any right triangle built out of its components.

Principle 3: Whenever you have a problem involving vectors, you should:

1. Break them into components.
2. Separate the x-components and y-components so you can deal with them independently.
3. Do each of the series of analyses/equations separately for the x- and y-components.
4. Put the resultant x-component and y-component vectors back together using the *Pythagorean theorem for magnitude*, and one of the *trigonometric functions for direction*.

There is one exception to Principle 3, and that is when you are dealing with cross-products.

Cross products and dot products

There are two ways to multiply vectors: dot products, and cross products. *Dot products* are based on the parallel components—to find the dot product you need to multiply just the parallel components of the two vectors being multiplied. The result of a dot product is a *scalar*—it has no direction, just magnitude. *Cross products* are based on the perpendicular components, and the result of a cross product is a new vector—it has both direction and magnitude. To find the *magnitude* of a vector from a cross product you must multiply the perpendicular components of the vectors you are multiplying. The direction of the resulting vector will actually be perpendicular to the two vectors you are multiplying (e.g., if you found the cross product of a vector on the x-axis and a vector on the y-axis, the resulting vector would be on the z-axis), and is given by the *right-hand-rule* (which you will learn in class, and is not covered here). Most of the time you will be using dot products, but you will need to understand both and the difference between the two.

A simple example of a case that requires a dot product is pushing a box across the floor. You apply a force in the same direction you want the box to move, and work would be calculated by simply multiplying the component of this force that is parallel to the floor, and the distance the box is moved across the floor. A simple example of a case that requires a cross product is using a wrench to unscrew a bolt. You must apply a force perpendicular to the length of the wrench, and the torque would be calculated by multiplying the component of the force perpendicular to the wrench by the length of the wrench.

Signs, positive and negative

The sign of a number, whether it is positive or negative, can have many different meanings, and I will introduce you here to what they mean in mechanics. In terms of scalars these just have the usual significance, namely whether you add or subtract a quantity. In order to understand their meaning in vectors, think about the relationship of positive and negative to quantities on a standard number line. Adding a *positive* number (or subtracting a negative number) is equivalent to *moving to the right,* while adding a *negative* number (or subtracting a positive number) is equivalent to *moving to the left*. On the number line, then positive and negative can be said to correspond to *direction.*

This is exactly the meaning that positive and negative will have in relation to vectors, you will *add x-component vectors that point right,* and *subtract x-component vectors that point left.* Likewise, you will *add y-component vectors that point up,* and *subtract y-component vectors that point down.* It doesn't actually matter whether you use these sign conventions (positive = right/up, and negative = left/down) as long as you are consistent, however these are the *standard conventions*, and it will be easiest if you adopt them because then your answers will be consistent with what your graders are looking for.

One last word

Students new to physics usually want to plug their numbers into the equations as soon as they get them on paper. This is a very bad habit to get into. You should not plug in the values for your variables until you have the solution written as a formula at the end of the problem. There are several reasons. First, it makes your math transparent, which means that if you make a mistake it is easier to go back and fix, as well as to double check your work. Second, it also makes your math transparent for your grader so you will get *more partial credit* this way if you do not get the right answer. Third, you may realize that you are missing some important information that is crucial to solving a problem. Fourth, it is just a good habit not to rely on your calculator to do everything for you. It may seem a little more difficult to do this at first, but start early and you'll thank yourself later!

Physics Concepts

Physics Concepts

Human intuitions

We have a number of intuitions about physics that are part of human nature and human understanding. In many ways they are very good; for example an outfielder knows intuitively how to catch a fly ball. However, in many ways our intuitions can be misleading. It is helpful to know what intuitions we must overcome to see why the beginning student (and many advanced students) make mistakes thinking about physics.

Our intuitive understanding of physics leads us to believe that the natural state of objects is to be at rest. When they are moving, our minds think it is because they have a sort of "impetus" or internal energy that propels them forward. As they move this energy or impetus peters out and they return to their natural state, rest. In contrast to this model, Newtonian physics says that the natural state of objects is constant velocity, and that the state of rest is just a specific case in which the constant velocity just happens to equal 0. Objects move with a constant velocity, which only changes due to an acceleration, and therefore a net force. Keep this in mind: *constant velocity is the rule—everything moves at constant velocity unless a force acts upon it.*

Because velocity is a vector that has both magnitude (speed) and direction, a force is needed to change *either* the magnitude, or the direction of velocity. Our intuition is quite good at helping us understand forces that affect the magnitude of a velocity. Imagine throwing a ball against a strong wind versus with a strong wind. The ball thrown with the wind will obviously travel faster than the one thrown against the wind. Sometimes we have a decent intuition for how constant forces change the direction of velocity. Imagine throwing a ball against a strong cross wind. The ball obviously changes direction; in this case the magnitude of the velocity will change as well. However, when we think about forces that *only change the direction* of the velocity, our intuitions often get us into trouble. A force that only changes the direction of a velocity is called a centripetal force and results in rotational motion. We usually experience this in cases such as going around a corner in a car.

Our problem is that our intuitions are based on an egocentric point of view when we are in rotational motion. We feel as though there is a "centrifugal force" pulling us outwards when we are moving in any kind of circle. Centrifugal force is an illusion created by using an accelerating reference frame. What actually happens as you go around a corner in a car is that your body tries to keep going straight (constant velocity), while the car is turning (velocity is not constant because it is changing direction); something has to put a centripetal force on you (like a car door) to keep you moving in a circle with the car. We experience this as though we are being pulled outward, when in fact we are just trying to go straight and are actually being pulled inward.

Our intuitions tell us that heavy objects fall faster than light objects—a bowling ball will fall faster than a piece of paper. While this is true, it is only because there is more air resistance slowing the piece of paper down as it falls. If you crumple the paper into a ball, it will actually fall just as fast as the bowling ball. Mass does not affect the rate at which things fall.

Remember that we have many biases from our evolutionary past that lead our intuitions astray when thinking about much of physics. Be aware of these, and it will help you avoid many common pitfalls, as well as help you develop a real intuition for what is going on in physics.

Kinematics

Basic quantities

Kinematics is where the real physics starts. There are four quantities that interact in the kinematic equations: *time, distance, velocity,* and *acceleration*. Distance, velocity and acceleration are vectors, whereas time is a scalar. Furthermore, in order for the kinematic equations to hold, the *acceleration must be constant*.

Kinematics quantities: *acceleration must be constant*

1. Time (t), standard units = *seconds* (s)
2. Distance (x or y), standard units = *meters* (m)
3. Velocity (v), standard units = *meters per second* ($\frac{m}{s}$)
4. Acceleration (a), standard units = *meters per second squared* ($\frac{m}{s^2}$)

*Distance is often represented by the variables "d" or "s" as opposed to "x" or "y" as noted above, I use x and y because these correspond to the x and y axes to help keep my vectors straight.

Hopefully if you look at the units you can see relationships between these quantities. For example: velocity is the change in distance divided by the change in time, or $v = \Delta x/\Delta t$, and acceleration is the change in velocity divided by the change in time, or $a = \Delta v/\Delta t$. Check the units here for unit analysis as discussed in the math intro. "Δ" (pronounced 'Delta') simply means "change in". For any quantity, it's just the final value minus the initial value, i.e.. $\Delta x = x_{final} - x_{initial}$

Principle 3 revisited: Velocity and acceleration are vectors

Both velocity and acceleration are vectors, so one must be careful when working in more than one dimension. If you have two vectors that are not parallel with each other, then they must both be broken into components, and you will solve for the x- and y-components separately. By separately, I mean SEPARATELY! An acceleration that is only in the y-direction (i.e. up or down, like gravity) *will not* affect any velocity in the x-direction. This will be very important in trajectory problems, as demonstrated below.

Kinematics equations

Kinematics equations will only work with a *constant acceleration*. What if there is no acceleration? That would just be a constant acceleration of zero ($a = 0\ m/s^2$). Kinematics problems are usually straightforward word problems: you simply need to figure out which variables you know, what variable you are looking for, and then what equation contains those variables. The kinematic equations provide a foundation for later concepts, so it's worth taking the time to get comfortable with them and make sure you know them well enough that you don't forget them.

Other than the definitional formulas presented above ($v = \Delta x/\Delta t$, and $a = \Delta v/\Delta t$) there are four kinematics equations that you will need to be familiar with:

1. $v_f = v_i + at$
2. $x_f = x_i + v_i t + 1/2at^2$ which can also be written $\Delta x = v_i t + 1/2at^2$
3. $\Delta x = (v_f - v_i)\, t/2$ this is just average velocity multiplied by time
4. $v_f^2 = v_i^2 + 2a\Delta x^*$

*this equation will not work if the two velocities have a different sign (+ and -), so be careful!

A note on variables and conventions

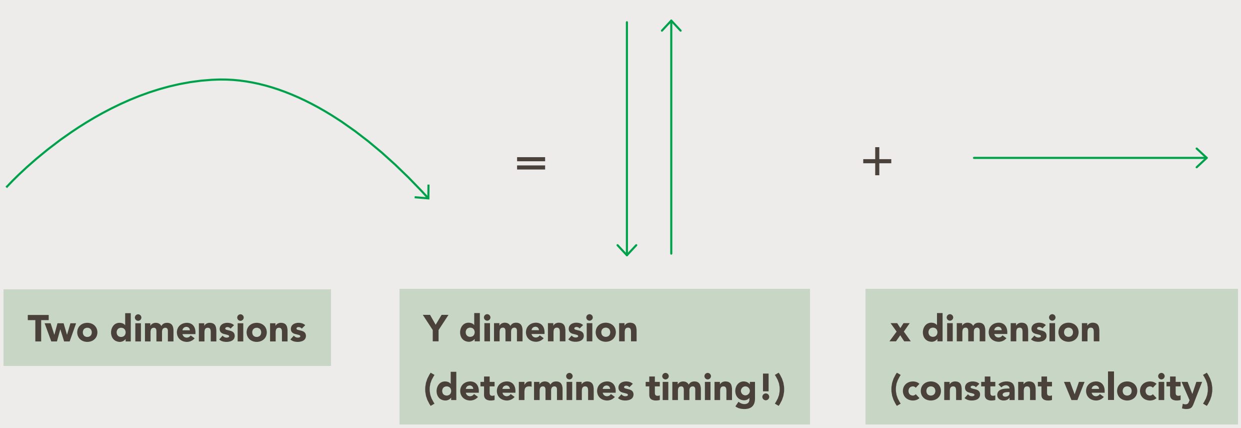
I have a specific set of labels that I use (e.g., "x" for position). While these are pretty much standard, they inevitably vary from class to class and teacher to teacher. I have labeled and named my variables above, but have not yet talked about subscripts. Often in problems there will be more than one velocity, say a final velocity and an initial velocity as above. In this case I recommend using subscripts to keep your variables clear and separate. I use a subscript "i" for initial, and "f" for final, but I could just as easily use "1" for initial and "2" for final. So, above v_i = initial velocity, and v_f = final velocity.

Principle 4: Make sure you know what every variable represents, and its units. If you don't know what a variable is in an equation, look it up!

Trajectory problems

A trajectory problem is a problem where an object starts out with some velocity at an angle, and will thus carve out a curved trajectory. Many students struggle with these problems because they do not understand vectors. The big mistake they make is forgetting to break the vector into components, making it difficult to deal with gravity. In trajectory problems an object is accelerating only in the y-direction! The acceleration in the x-direction is zero. You can treat these directions as completely independent.

The trajectory ends when the object hits the ground, and so the y-component determines how long the object will be in the air. In fact, a trajectory can be thought of as two trajectories: one that goes straight up and down, and one that goes straight in the x-direction at constant velocity.



To solve trajectory problems:

1. Break *your velocity vector into components* using your trig functions as seen in the diagram below
2. Solve for each component *separately* using your kinematic equations as if the object was just moving in one dimension
3. Solve for the *y-component first*, usually solving for time, which you can then plug into the x-component kinematics. Remember that your acceleration in the y-direction is *always* gravity (if there are not other forces involved) and thus $a = g = -9.8\ m/s^2$
4. Solve for your x-component by using the time from the y-component. As long as there is no other force there will be no acceleration in the x-direction, as gravity only acts in the y-direction. The distance the object goes in the x-direction is called the *range*.

For example say we are given the following problem. A batter hits a baseball with an initial speed of 25 m/s, at an angle of 60° from the horizontal. What is the range of this ball?

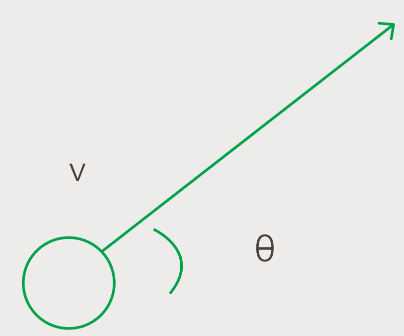
1. We should define our variables and draw a picture:

$v = 25\ m/s$

$\theta = 60°$

$a_x = 0\ m/s^2$

$a_y = g = -9.8\ m/s^2$



2. Now that we are set up we need to break this vector into components so that we can do our algebra.

    $v_{xi} = v(\cos\theta)$

    $v_{yi} = v(\sin\theta)$

3. Only now can we jump into the kinematics equations. We will start with the y-component because that will give us a time to plug in for the x-component. There are two ways to do this. The first option is to break the trajectory in two pieces, in which case $v_{yf} = 0$ at the top of the trajectory for the first piece. We can solve for the first piece and simply double it (because the rise and fall are symmetrical, they will take the same amount of time). Or, since we know that the rising piece is the exact opposite of the falling piece, we can assume that at the ground $v_{yf} = -v_{yi} = -25$ m/s ($v_{yi} = 0$ for the second piece because it starts at the top of the trajectory).

I will use the first method.

$v = 25$ m/s

$\theta = 60°$

$v_{iy} = v(\sin\theta) = 21.6$ m/s

$a_y = g = -9.8$ m/s^2

$$v_{fy} = v_{fy} + a_y t$$

$$0 = v_{iy} + gt$$

$$-v_{iy} = gt$$

$$t = \frac{-v_{iy}}{g} = \frac{-21.6 \text{ m/s}}{-9.8 \text{ m/s}^2} = 2.2\text{s}$$

This is half the trajectory so we multiply by 2 to get t = 4.4s

$v_{ix} = v(\cos\theta) = 12.5$ m/s

$t = 4.4$s

$a_x = 0$ m/s^2

$$\Delta x = v_{ix}t + 1/2 a_x t^2$$

$$\Delta x = v_{ix}t + 0 = v_{ix}t$$

$$\Delta x = (12.5 \text{ m/s})(4.4\text{s}) = 55\text{m}$$

$$\Delta x = 55\text{m}$$

Forces

$$\vec{F} = m\vec{a}$$

Forces are vectors

There are a number of vectors you will need to deal with in physics, but vectors will be the most important when you deal with forces. Every time you have more than one force acting on any object these forces must be added together, and they will add together as vectors. This means that you should get in the habit of ALWAYS breaking every force into x- and y-components as soon as you want to do any calculations with it. You will then add together all of the x-components (with right-facing vectors as positive, and left facing vectors as negative), and then *separately* add together all of the y-components (with upwards-facing vectors as positive, and downwards-facing vectors as negative).

Principle 3 revisited: Forces are vectors, so any time you are dealing with forces you will break them into x- and y-components, and add these separately. To get the magnitude of the final vector you will recombine these results using the Pythagorean theorem, and to get the direction of the final vector you will need to use trigonometric functions.

Forces are accelerations

Probably the equation that will have the single biggest impact on your grade in physics is the basic equation for a force. A simple rearrangement of this formula shows us that mass is just a measurement of how much acceleration a given object will experience per unit of force. In other words, *any time you have a net force you have an acceleration*. This means that if you add up all of your force vectors and get a resulting force that is not 0 then your object will be experiencing some sort of acceleration.

Forces: causes and descriptions

There are many types of forces: electrical, gravitational, frictional, drag, etc. Whatever the source of a given force, it will cause an object to accelerate either in a line or in a circle. These are *descriptive* equations—they describe the effects a force has. However, there are also *causal* equations of force—these describe what creates a force and how strong it is. An example of this second type would be gravitation: *Causal equations* tell you the strength and direction of a force, whereas the descriptive equation tells you what the force does to the object. Usually you can set up an equation linking the two and never calculating the actually force F.

$$F_{causal} = F_{descriptive}$$

$$M \times g = M \times a$$

$$g = a$$

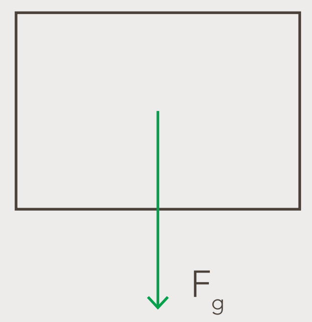
It takes two bodies to make a force. This is very important, because it tells us that a force cannot come from nowhere. At the beginning you will just be given values for forces, but later in the course you will use other equations to actually solve for the magnitude of the force applied by gravity, or friction, etc. (this is what was done above).

Superposition and free body diagrams

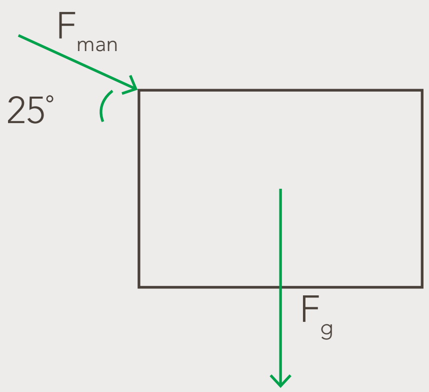
Because forces are vectors they must be added as such, and the easiest way to get started so that you can see what forces you should add together and how, is to draw what is called a *free body diagram*. This is just a picture of the object of interest on its own, with all the force vectors drawn in. Creating a free body diagram is very similar to defining your variables, you just need to figure out what the forces are acting on your object, what each one's direction is, and then draw them in.

For the following problem we are going to draw a free body diagram.

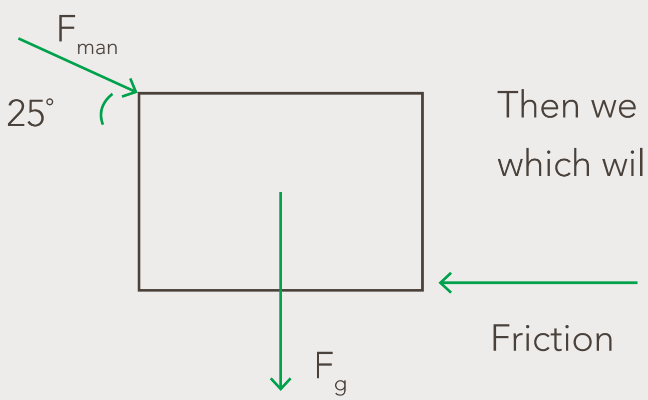
A man moves a box by pushing it across the floor. Because of his height, he pushes the box forward and down, with his force making a 25° angle with the horizontal. The floor has not been waxed in awhile, and so there is a strong frictional resistance to his push. Draw a free body diagram.



First we draw our box, and a vector for gravity



Next we will draw in the man's pushing force



Then we add in the friction force, which will always oppose motion

This should have all of the vectors we are dealing with. Now we can just add them together like any other vectors.

After you do your free body diagram you will break the vectors into components as usual, and then do regular vector math, keeping your x- and y-components separate, and recombining using Pythagorean theorem and trig functions.

Equilibrium problems

There is a special class of force problems called equilibrium problems. An object is "in equilibrium" when its velocity is constant. If the velocity is constant, what does that mean about the net force on the object? The only way to have a constant velocity is if all forces sum to 0. You will no doubt use this information to solve for one of the forces. In order to do so you will create equations for forces as usual, but rather than setting them equal to "ma", you will set them equal to 0. Often you will have multiple equations here for the different dimensions and so will have to use the substitution method or the elimination method to solve for all the unknowns.

Equilibrium Problems

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma F_z = 0$

$\Sigma \text{Torques} = 0$

In order to solve an equilibrium problem:

1. Draw a free body diagram
2. Break all forces into components
3. Set up force equations, and set all of them equal to 0
4. Solve for all unknowns, making sure you have as many equations as you do unknowns

Centripetal forces

A centripetal force is any force that *acts perpendicular to an object's velocity*. Velocity has both magnitude and direction, and until now we have been discussing forces that change the velocity's magnitude. Centripetal forces instead change *only* the direction of the velocity. Centripetal forces cause bodies to "orbit" the way a planet does around the sun. Have you ever spun a ball around on a string? The tension you feel in the string keeps the ball moving in a circle, and you create this tension with the centripetal force of your hand. If you were to let go, the ball would fly off in a straight line, a line tangent to where it flew out of the circle.

When dealing with centripetal motion, remember that the equation $F_C = mv^2/r$ is the "ma" from Newton's Second Law. In other words, all the forces acting on a body in circular motion with speed v must add up to equal mv^2/r.

For example to find the radius of an orbiting body, we set the descriptive formula $\left(F_c = \frac{mv^2}{r}\right)$ equal to the formula for the source of the force, gravity $\left(F_g = \frac{G(m_1 m_2)}{r^2}\right)$ to get:

$$\frac{mv^2}{r} = \frac{Gm_1m_2}{r^2}$$

The masses cancel ($m = m_1$)

$$\frac{v^2}{r} = \frac{Gm_2}{r^2}$$

Multiply r across

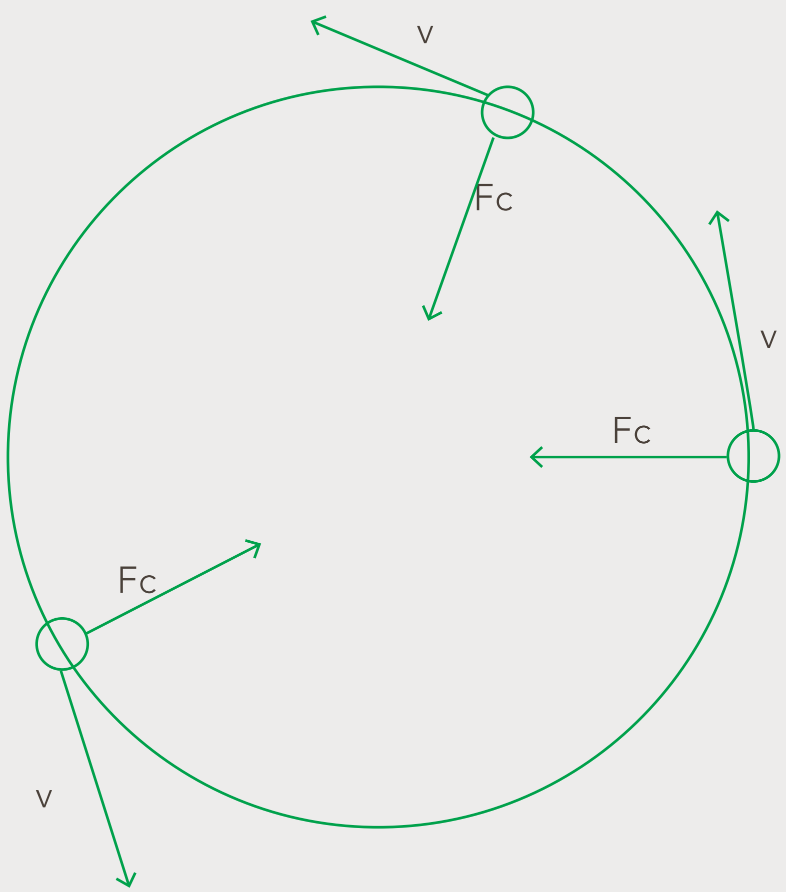
$$r = \frac{Gm_2}{v^2}$$

Multiply r to the other side, and divide v^2 over

$$\boxed{r = \frac{Gm_2}{v^2}}$$

Draw a box around your answer

Remember that a centripetal force will always be exactly perpendicular to the velocity. The big circle below is the line of motion; everything else is labeled either velocity or centripetal force. Notice how the two arrows are always perpendicular. Notice also how the problem above was solved with just variables. Once you get the equation at the end, then you can plug in your numbers.



Energy

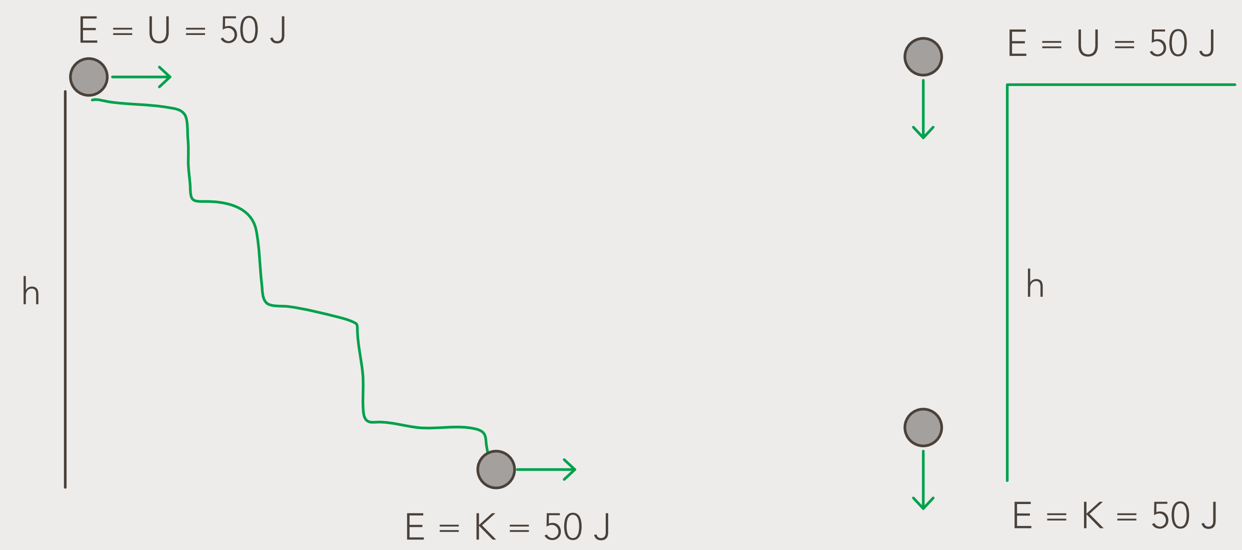
The mystery of energy

Energy is probably the most important single idea in understanding physical processes. If you find yourself confused about energy, that's ok; it's hard to define. However, we do know how to measure it, and how it behaves. The important thing to remember is that though energy comes in many forms, and can be transferred from one form to another, *energy is always conserved.*

Energy is a scalar

Energy is not a vector like force, velocity, and acceleration. This means that it is *path independent*. That is the technical way of saying that it does not matter how something gets there, the only important thing is where it is and what it is doing. For example a ball rolling to the right would have a positive velocity. The same ball going the same speed, but going left would have a negative velocity. However, in both situations the ball would have the same amount of energy, $K=1/2mv^2$.

Because the velocity vector is squared, the *directional information is lost and kinetic energy is always positive* (since directional information was stored in the sign of the velocity vector). This is helpful when solving many kinematics problems. Whereas before we needed to know distance, time, and acceleration to find the velocity, now we can simply use energy. Here is a little illustration. Notice that the path the ball takes is irrelevant to the energy, and therefore to the velocity as well.



In the above diagram K represents kinetic energy and U represents potential energy

Because this is the same ball, and thus has the same mass, both of these have the same amount of kinetic energy at the bottom and therefore both have the same speed (although not the same velocity because they are moving in different directions)

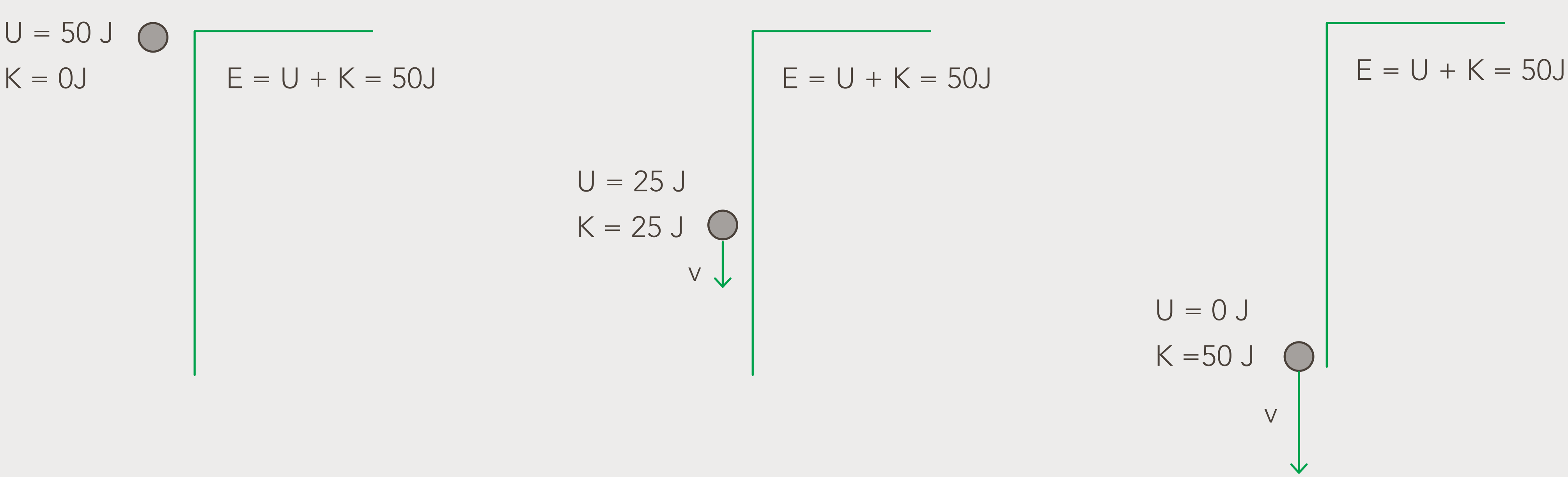
Conservation of energy

You will learn that energy is always conserved. You will also learn that "it can neither be created nor destroyed." These are the technical phrasings, and we are going to unpack them a little here. To say that energy is conserved means that *it can change forms, but you must always finish with as much as you start with*. This is just like mass, which is also conserved. You can take an object and take matter from it (think of taking a piece out of a cake), but that matter is now just another object (the piece of cake on your plate), in a new form (you can eat it but then it is just broken up into its chemical components).

You can't make matter disappear; you can only make it change form. For example, a ball on a cliff has gravitational potential energy (let's say 50 Joules). When the ball rolls off the cliff, it will lose its potential energy. But for every Joule of potential energy it loses, it will gain a Joule in kinetic energy. It speeds up! At the bottom of the cliff, right before the ball makes impact with the ground, it has lost all of its potential energy. Then we know it must have 50 Joules of kinetic energy; it's going really fast now! That's the meaning of conservation of energy. *Energy has changed forms, but we always have the same amount we started with.*

Total Energy is K and U so E = K + U

$$U = mgh \text{ and } K = \frac{1}{2}mv^2$$



Energy’s many forms

You will learn about many different forms that energy can take. As problems become more complicated, it’s important that you become an “energy auditor” making sure every last Joule is accounted for. If energy ever seems to disappear, look for it to reappear elsewhere (except in problems that explicitly lose energy such as *inelastic collisions*). Energy equations are listed below in the order that you will encounter them through the course.

Notice the fourth equation down is E = W = Fd. W is work, and this equation is the foundation of the *Work-Energy Theorem*, covered below. F is force, and can be any force, and d is a distance. So energy is just a force applied over a distance as discussed below, and this equation couples energy and forces. Just as acceleration couples together forces and kinematics, work couples together energy and forces.

Energy Equations

$K = \frac{1}{2}mv^2$	**Kinetic Energy**
$U_g = mgh$	**Gravitational Potential Energy**
$U_s = \frac{1}{2}kx^2$	**Spring Potential Energy**
$E = W = Fd$	**Work - Force Relationship**
$K = \frac{1}{2}I\varpi^2$	**Rotational Kinetic Energy**
$U_g = \frac{G(m_1m_2)}{r}$	**Gravitational Potential Energy**
$E = Pt$	**Power - Energy Relationship**
$U_e = qV$	**Electric Potential Energy**
$U_e = \frac{K(q_1q_2)}{r}$	**Electric Potential Energy**

The Work-Energy Theorem: connecting energy and forces

The work energy theorem states that *the work done by the net force on a particle equals the change in the particle's energy*. This means essentially that work is energy. Work is important in two major ways:

1. It provides an interface between forces and energy, just like acceleration does with forces and the kinematics
2. It shows how we can put energy into a system (like pushing a ball up a hill), or take it out (like losing energy to the resistance of friction).

The following equations are definitional for the work energy theorem

Work Energy Theorem

$$W = Fd$$

$$W = \Delta E$$

$$\Delta E = F\Delta d$$

Revised Energy Equation

$$E = K + U + W$$

Momentum

$$\vec{p} = m\vec{v}$$

P is for Problems

Momentum is one of the easiest concepts in general physics, but it leads to many common pitfalls. The first major issue is that the variable that represents momentum is P, as shown above. Makes sense, right? (no). "P" seems to have no connection to momentum unlike most variables that are just the first letter of the word. Worse, there are a number of *other* quantities labelled "P": *Power* (Energy per unit time or E/t, or E = Pt); *Pressure* (Force per Area). Fortunately neither of these are vectors, and so if you have a P *with the vector arrow above you can know that it is momentum.*

Momentum is a vector

Momentum is a vector just like forces, velocities, accelerations, etc. This means that momentums must be added as vectors, which will be especially relevant for doing problems on collisions. Furthermore, this is very important to keep track of in problems in which velocity changes direction, *don't forget your signs here*. For example if the initial velocity is 15 m/s to the right or v_i = 15 m/s, and the final velocity is 15 m/s to the left or v_f = -15 m/s then $\Delta v = v_f - v_i$ which is -30 m/s, *not* 0 m/s. This is a common mistake and an easy one to make.

Momentum, impulse, and the relationship to force

You have learned to use acceleration to describe motion, but originally Issac Newton used momentum instead. He stated in terms of what we now call an *impulse*, which is denoted by the variable J. J is the change in momentum, and is also equal to a force multiplied by the time it is exerted.

$$\vec{J} = \vec{F}\Delta t = \Delta\vec{p}$$

This is not usually a major topic, and is typically introduced to explain momentum, you get one homework problem, and then you get the inevitable question on the test about it. This is a very easy concept and should be free points, but everyone always forgets about it come test time.

There are very few derivations in this book but the following one is easy to understand, is historically important, and is a simple demonstration of how really understanding equations can be helpful.

Newton's original formulation of his force was actually about changing momentum.

$F = \frac{\Delta p}{\Delta t}$ **This was his original formulation (except it was calculus)**

$p = mv$ **This is the definition of momentum**

$F = \frac{m\Delta v}{\Delta t}$ **Just substitute $m\Delta v$ for p**

$a = \frac{\Delta v}{\Delta t}$ **This is the definition of acceleration**

$F = ma$ **Just substitute a for $\Delta v/\Delta t$**

Can it be this simple?

The concept of momentum itself is straightforward; however, momentum problems can quite difficult, especially collision problems. Collision problems are not conceptually difficult; they usually just have a lot of moving parts. With a little practice, these should be easy to master. This is your chance mid-semester to take a little breather, as long as you know how to solve equations with multiple unknowns, and are very good with your vectors (yet another reason to master such concepts early on). You will want to use a lot of drawings and then just do your vector math.

Principle 3 revisited: Momentum is a vector, so make your drawing first and immediately break all vectors into components. Do the math on the x-components, do the math on the y-components, and then put the resultant vectors back together using trig functions and the Pythagorean theorem. Beware of the signs, positive and negative vectors can cause careless mistakes!

Collisions: the confluence of energy and momentum

A collision problem is exactly what it sounds like, two objects with energy and momentum colliding. There are two fundamental types of collision problems: *elastic* and *inelastic*. There are three types of problems you will be solving though because inelastic can be broken down further into a subcategory of *perfectly inelastic*. In an *elastic collision* both energy and momentum are conserved. In an *inelastic collision* momentum is conserved but energy is not. A *perfectly inelastic collision* is just like an *inelastic collision*, with one added condition—the objects stick together. This is all very important to keep track of because it will dictate how we do our problems, and it is summarized on the table on the next page.

Type of Collision	What is conserved	What equations you can expect to use
Elastic	Momentum is conserved	$p_1 = p_f$ $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_1v_{1f} + m_2v_{2f}$
	Energy is conserved	$E_i = E_f$ $\frac{1}{2}m_1(v_{1i})^2 + \frac{1}{2}m_2(v_{2i})^2 = \frac{1}{2}m_1(v_{1f}) + \frac{1}{2}m_2(v_{2f})^2$
Inelastic	Momentum is conserved Energy is NOT conserved	$p_1 = p_f$ $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ $E_i \neq E_f$
Completely Inelastic	Momentum is conserved Energy is NOT conserved (also objects stick together)	$p_1 = p_f$ $m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)\,v_f$ $E_i \neq E_f$

You will notice that I write $p_i = p_f$, and $E_i = E_f$ above the equations where these are elaborated and expanded, this is a good technique, as these simple equations mean "momentum is conserved" and "energy is conserved" respectively (of course don't put in that energy is conserved in an inelastic collision problem). This will usually get you partial credit, even if you can't write anything else. The lesson here is to always put down what you know in any problem, even if you can't get further, you can usually still squeeze out a few points.

Rotation

Principle 5: Rotation is just like everything else, but in a circle. This sounds ridiculous at first but you will realize that there is a rotational analog of everything you have learned so far, there is angular displacement, angular velocity, angular acceleration, angular forces (called torques), angular kinetic energy, angular momentum, and so on. Once you understand how similar everything is, the complexity should just melt away.

From linear dynamics to rotational dynamics and back again

There are two factors that you need to understand inside and out to understand how linear dynamics corresponds to rotational dynamics: (1) how circumferences, distances, and radians are related; and, (2) what a moment of inertia is. Once you understand these two concepts and work with them a little bit you should see that there are really very few differences between rotational dynamics and linear dynamics, even all the equations are basically the same. If you can wrap your mind around all of this, you can simply transfer all of your existing knowledge onto rotation and save yourself a lot of time and angst.

The first piece of the rotational puzzle:
Circumference, distance, and radians

The entire reason that we reviewed circumference and radians so extensively in the math review is because it is the key we need to unlock the rotational dynamics equations. We are going to start by thinking about a tire on a unicycle and build everything from that. If you had a unicycle tire with a radius of one meter, how far forward would you have traveled if the tire went around once? The easiest way to conceptualize this is to first realize that every part of the tire will touch the part of ground that it goes over, so it is covering the same amount of distance forward as the length of its circumference.

Our tire has gone the distance of its circumference in one full rotation, twice the circumference in two and so on

$$C = \pi \qquad \text{Distance} = 2\pi\, d$$

Because d = 1, then c = π which is 3.14

Through this very simple demonstration we can see that our distance is always going to be the number of rotations multiplied by the circumference. If the circumference is 2πr, then we can also see that our distance traveled will always be (2π)(# of rotations)(r).Wouldn't it be convenient if we could just set 2π equal to one rotation, so we could call one rotation 2π, two rotations 4π, and three rotations 6π, instead of 360°, 720°, and 1080°? It would indeed. In fact, that that is exactly what radians accomplish. Recall that in radians 2π = 360°. You may also remember that radians are said to be a "unit-less" number: they don't need units, like meters or degrees. This is because they are a conversion factor between rotational distance and linear distance. As long as we measure the angle of rotation θ in radians then our linear displacement along the circumference of a circle, x will just be our radius times θ.

$$\Delta x = r\theta$$

This insight provides the backbone for understanding rotational dynamics based upon what you already know about linear dynamics. θ is analogous to displacement in rotation, in fact it is called our *angular displacement*. This behaves just like displacement, x, does in our linear equations, but is just for rotational kinematics.

Rotational analogues

All of our rotational variables and quantities have linear analogues, and here they are:

Linear quantity	Rotational analogue
Displacement (x)	Angular displacement (θ)
Velocity (v)	Angular velocity (ω)
Acceleration (a)	Angular acceleration (α)
Mass (m)	Moment of inertia (I)
Force (F)	Torque (τ)
Momentum (p)	Angular momentum (L)
Kinetic energy ($K = 1/2mv^2$)	Kinetic energy ($K = ½ I\omega^2$)

Rotational kinematics

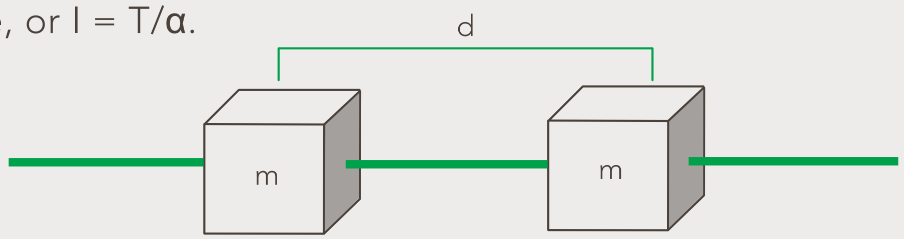
Every single equation in kinematics has an analogue in rotational kinematics. First I want to show you another quick derivation so you see why these are all essentially the same equations as the linear kinematics.

Deriving rotational kinematics equations from linear kinematics equations	
$V_f = V_1 + at$	We start with one of our basic kinematic equations
$\frac{V_f}{r} = \frac{V_1}{r} + \frac{at}{r}$	We divide both sides by r. We know that v = ωr and a = ar so, ω = v/r and α = a/r
$\omega_f = \omega_1 + at$	We arrive at one of our basic rotational kinematic equations we can always go back by just multiplying everything by r

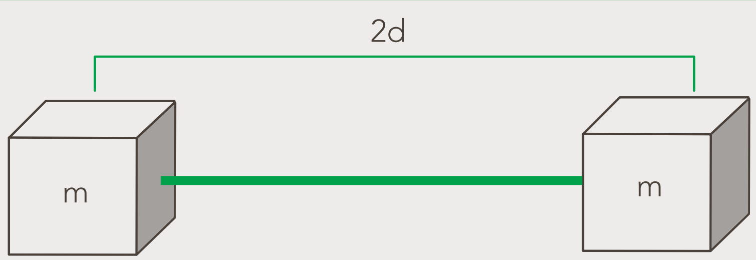
You will see that all the kinematic equations have a direct rotational analogue. You can easily get back and forth by simply multiplying or dividing by the radius.

The second piece of the puzzle: moment of inertia (I)

In order to understand moment of inertia we must first take a closer look at what exactly mass is. We know that F = ma, or better for our current purposes, m =F/a. That is, mass is essentially a measure of *how much acceleration we get per unit of force*. Think about what this means, it is essentially the amount of inertia that an object has, *the amount that object resists change*, since the normal state of matter is constant velocity. Therefore, *Mass is a measure of an object's inertia*. It should come as no surprise then that the rotational analogue of mass is called a *moment of inertia*. By analogy, then, the moment of inertia, I, is a measure of how much angular acceleration will be produced by a torque, or $I = T/\alpha$.



Total Mass = m+m=2m
Moment of Inertia (I) = $m(d/2)^2 + m(d/2)^2 = 1/2\ md^2$



Total Mass = m+m=2m
Moment of Inertia (I) = $m\ d^2 + m\ d2 = 2\ md^2$
(4 x above!)

This is because for rotation, it is not just the amount of mass that is important, but how far the mass is from the axis of rotation. Moment of inertia takes this into account: *the further mass is from the axis of rotation, the greater the moment of inertia.* The equation for moment of inertia is:

$$I = \sum mr^2$$

Why moment of inertia?

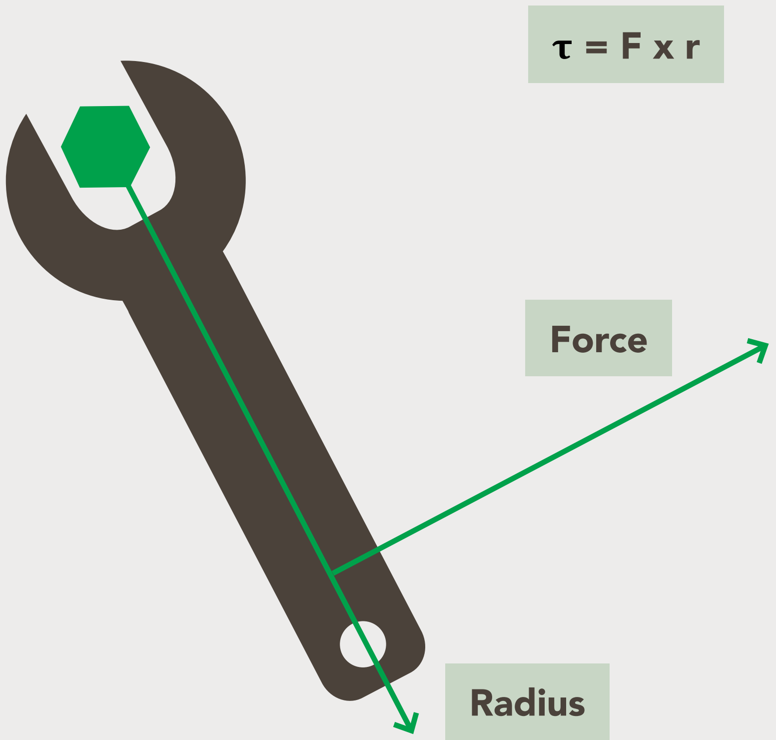
The amount that an object resists linear change is merely dependent upon *how much mass it has*, but the amount that an object resists rotational change is dependent upon *not just how much mass the object has*, but also *how that mass is distributed*. This can be understood using a simple example of weights on a barbell. Imagine I place two weights on a bar very close together. As a workout, I lift the weights, and then spin the weights around in a circle. It's a pretty good workout. Then I move the weights to the end of the bar. Again I lift the weights, which feels exactly the same. However, when I try to spin the weights, I can barely do it!

Torque (*τ*)

Torque is probably the most difficult topic in rotational dynamics. Torque is the rotational analogue of force. Once you understand moment of inertia conceptually, torque is not that hard to conceptualize, except that it has one new unique feature: a cross product. We'll get to that below.

Just as the distribution of mass is irrelevant in linear dynamics, so is the exact place where a force is applied. All that matters is the net magnitude and direction of the force. If one pushes a box on the top, the middle, or the bottom, these are all exactly the same. However, just as distribution of mass makes a difference in moment of inertia, where a force is applied also matters in torque. Luckily, this is also easy to conceptualize intuitively. Imagine that you need to remove a lug nut from a tire: would you prefer to use a miniature monkey wrench, or a giant lug wrench? It would probably be impossible with a miniature wrench because the leverage arm is too short. This gives us the intuition that the *further from the axis of rotation a force is applied, the greater the torque*. Easy enough, this is nicely captured in the torque equation to the right.

Torque is the first place that we really deal with a cross product. Think of it as a way to multiply vectors wherein the maximum result when they are perpendicular to each other. Try opening a door and thinking about torque. How and where do you push the door open? You want to push at an angle perpendicular to the door, and you could imagine that as the angle you push at becomes more parallel to the door, the harder you will have to push to open it.



Just remember that the cross product means that we multiply the perpendicular components of these vectors. You can do the following:

1. If the force is not already perpendicular to the radial arm then use the trig function such that: Torque = (F)(r)(Sinθ)
2. Take *only* the perpendicular component of the force, and then just multiply it by the radial arm as usual to get the magnitude of the torque
3. The direction of the torque vector is given by the right-hand-rule, something you will cover in class, but won't be covered here.

Conclusion

The goal of this book is to give you a conceptual framework to complement the technical material you will cover in class and in your course book. To accomplish this goal we have stripped out most of the technical math, and tried to get you ready for what you will see by walking through key math concepts in a non-technical way, and giving you a conceptual overview of important physics concepts. We hope that the math review is helpful, and will supply you with the tools you need to understand the math as it is applied to physics. We further hope that this book has helped you build up an adequate framework for you to hang the technical concepts on when you learn them, and has helped prepare you in a way to make them easier to learn and fully understand when you do come across them in class.

As you go through the course you may find it helpful to review the relevant material covered here before you learn each topic, and as you work your way through how best to understand each concept. Physics is often thought of as a very difficult course, and no matter how you go about learning physics it will certainly require a lot of work. But, if you work to understand the concepts, rather than just trying to memorize how to solve specific sets of problems, the class can actually be quite enjoyable, you will get more out of it with less effort, and you will be prepared to solve any problem your professor can throw at you, not just the specific problems you have worked through and studied. We hope this book helps you on your way, and best of luck in the class!

Of course, if you need any additional help, please visit *www.veritutors.com* or contact us at *info@veritutors.com* or 1-877-265-6582.

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